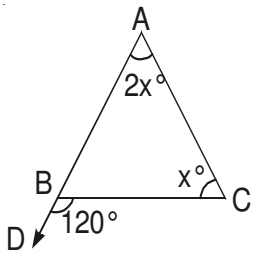
15 TRIANGLES, CONSTRUCTION OF TRIANGLES AND CONGRUENCY OF TRIANGLES

1. In each of the following figures one sides of a triangle has been produced. Find all the angles of the triangle in each case.

(*i*)



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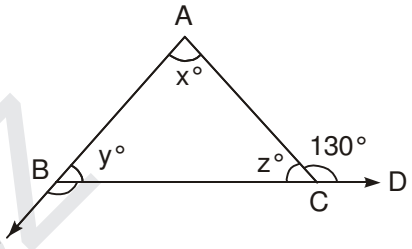

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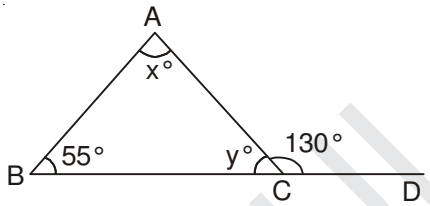


Ans. (*i*) Exterior angle = sum of opposite interior angles

$\Rightarrow$ $120° = 2x + x = 3x$, $3x = 120°$ $x = \frac{120°}{3} = 40°$

Thus, $\angle A = 2x = 2(40) = 80°$

$\angle B = 180° - 120° = 60°$ [Linear pair of angles]

and $\angle C = x = 40°$

(*ii*) Exterior angle = sum of interior opposite angles

$y = 180° - 140° = 40°$ [Linear pair of angles]

$z = 180° - 130° = 50°$ [Linear pair of angles]

$x + z = 140°$ $\Rightarrow$ $x + 50° = 140°$ $\Rightarrow$ $x = 140° - 50°$

$\Rightarrow$ $x = 90°$ $\angle A = x = 90°$ $\angle B = y = 40°$ and $\angle C = z = 50°$

(*iii*) Exterior angle = sum of interior opposite angles

$\Rightarrow$ $125° = x + 55°$

$\Rightarrow$ $x + 55° = 125°$

$\therefore$ $x = 125° - 55° = 70°$

$y = 180° - 125°$ [Linear pair of angles]

$y = 55°$

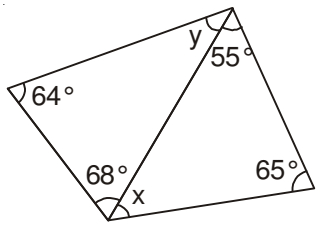
Thus $\angle A = x = 70°$

$\angle B = 55°$

and $\angle C = y = 55°$

2. Calculate the size of each lettered angles in the following figures :

(*i*)



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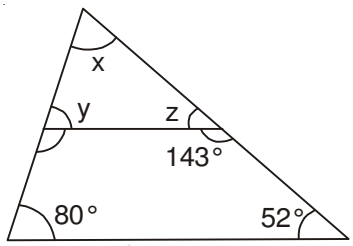


Ans. (*i*) $64° + 68° + y = 180°$ (Sum of angles of a triangle)

$\Rightarrow$ $132° + y = 180°$

$\Rightarrow$ $y = 180° - 132° = 48°$

Now, $55° + 65° + x = 180°$ (Sum of angles of a triangle)

$120° + x = 180°$

$\Rightarrow$ $x = 180° - 120° = 60°$

(*ii*) $z + 143° = 180°$ (Linear pair angles)

$\Rightarrow z = 180° - 143° = 37°$

Sum of interior angles of a quadrilateral = 360°

$\Rightarrow$ $p + 143° + 80° + 52° = 360°$

$\Rightarrow$ $p + 275 = 360°$

$p = 360° - 375° = 85°$

Now, $\angle p + y = 180°$ (Linear pair of angles.)

$85° + y = 180°$

$\Rightarrow$ $y = 180° - 85 = 95°$

Now, $x + y + z = 180°$ (sum of angles of a triangle)

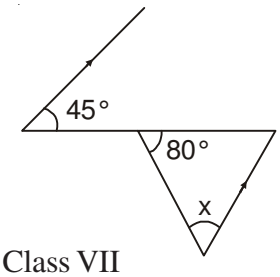
$\Rightarrow$ $x + 95° + 37° = 180°$

$\Rightarrow$ $x + 132° = 180°$

$\Rightarrow$ $x = 180° - 132° = 48°$

3. Calculate the size of each lettered angle in following figures :

(*i*)



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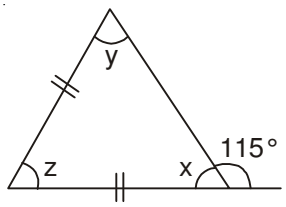


Ans. (*i*) Mark an angle y as shown

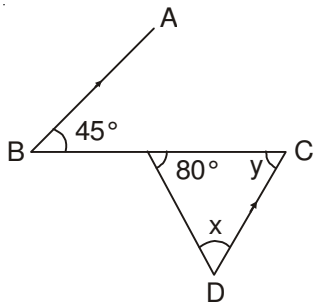
$y = 45°$ (alternate interior angles)

($\because$ AB || CD)

Now, $80° + y + x = 180°$ (sum of angles of a triangle)

$80° + 45° + x = 180°$, $125° + x = 180°$

$x = 180° - 125° = 55°$



(*ii*) Here $\angle y = \angle x$

$\because$ Angles opposite to equal sides.

$x + 115° = 180°$ (Linear pair angle)

$\Rightarrow$ $x = 180° - 115° = 65°$

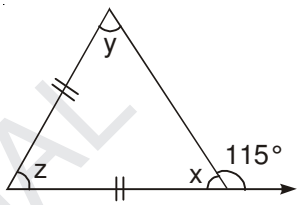
$\therefore$ $y = 65°$

$x + y + z = 180°$ (Sum of angles of a triangle)

$\Rightarrow$ $65° + 65° + z = 180°$

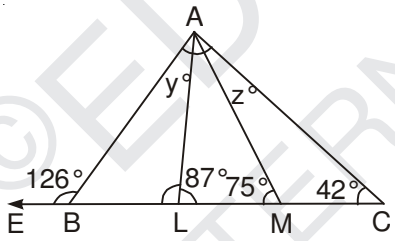
$\Rightarrow$ $130° + z = 180°$

$\therefore$ $z = 180° - 130° = 50°$



4. Calculate the size of each lettered angle in the following figures :

(*i*)



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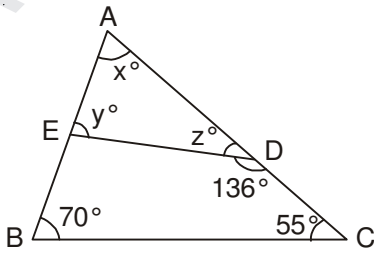


Ans. (*i*) Exterior $\angle AML = \angle MAC + \angle ACM$

$\Rightarrow$ $75° = z + 42°$

$z = 75° - 42° = 33°$

$\angle ABE + \angle ABL = 180°$ (Linear pair of angles)

$\Rightarrow$ $126° - \angle ABL = 180°$

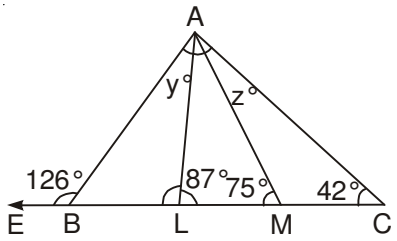
$\Rightarrow$ $\angle ABL = 180° - 126° = 54°$

Now in Δ ABL

Exterior $\angle ALM = \angle ALB$

$\Rightarrow$ $87° = 54° + y$

$\Rightarrow$ $y = 87° - 54° = 33°$



In ΔABL,

$\angle ABL + \angle BAL + \angle ALB = 180°$

$\Rightarrow$ $54° + 33° + x = 180°$, $87° + x = 180°$

$\Rightarrow$ $x = 180° - 87° = 93°$

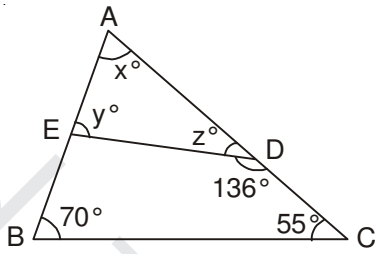
Hence, $x = 93°$, $y = 33°$ and $z = 33°$

(*ii*) In ΔABC $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\Rightarrow$ $x + 70° + 55° = 180°$ $\Rightarrow$ $x + 125° = 180°$

$\Rightarrow$ $x = 180° - 125° = 55°$

In ΔAED



Exterior $\angle EDC = x + y$

$\Rightarrow$ $136° = 55° + y$

$\Rightarrow$ $y = 136° - 55°$

$y = 81°$

But $\angle EDC + \angle EDA = 180°$ (Linear pair of angles)

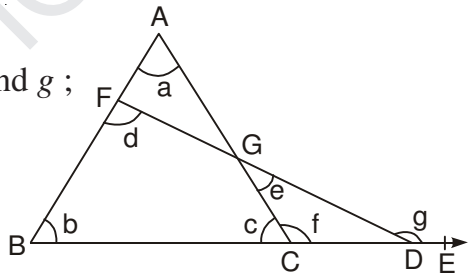
$\Rightarrow$ $136° + z = 180°$

$\Rightarrow$ $z = 180° - 136° = 44°$

Hence $x = 55°$, $y = 81°$, $z = 44°$.

5. Use the given figure to express:

(*i*) a in terms of b and f; (*ii*) e in terms of f and g ;



Ans. (*i*) In Δ ABC,

Exterior $\angle ACE = \angle A + \angle B$

$\Rightarrow$ $f = a + b$

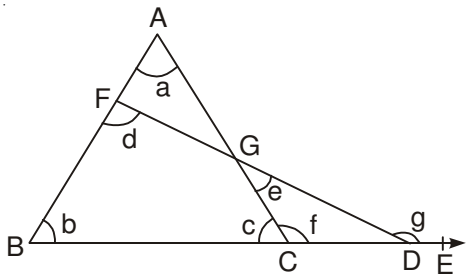
$\therefore$ $a = f - b$

(*ii*) In ΔCGD, Exterior

$\angle GDE = \angle DGC + \angle GCD$

$\Rightarrow$ $g = e + f$

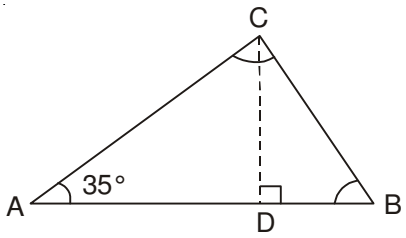
$\therefore$ $e = g - f$



6. In the adjoining figure, ABC is a right angle triangle right angled at C, and CD $\perp$ AB.

If $\angle CAB = 35°$, find :

(*i*) $\angle ACD$ (*ii*) $\angle ABC$



Ans. (*i*) $\therefore$ CD $\perp$ AB (Given)

$\therefore$ $\angle CDB = 90°$

Now, $\angle CDB + \angle CDA = 180°$ (Linear pair of angles)

$\therefore \quad \angle CDA = 180° - \angle CDB$

$= 180° - 90° = 90°$

In Δ ACD,

$\angle ACD + \angle CAD + \angle CDA = 180°$ (Sum of angles of a triangle)

$\Rightarrow \quad \angle ACD + 35° + 90° = 180°$

$\Rightarrow \quad \angle ACD + 125° = 180°$

$\angle ACD = 180° - 125° = 55°$

(*ii*) In Δ ABC

$\angle ABC + \angle ACB + \angle BAC = 180°$ (Sum of angles of a traingle)

$\Rightarrow \quad \angle ABC + 90° + 35° = 180°$

$\Rightarrow \quad \angle ABC + 125° = 180°$

$\therefore \quad \angle ABC = 180° - 125° = 55°$

7. Each base angles of an isosceles triangle is 15° more than its vertical angle. Find each angle of the triangle.

Ans. In an isosceles Δ ABC, AB = AC

Let vertical angle $\angle A = x$ then each base angle $= x + 15$

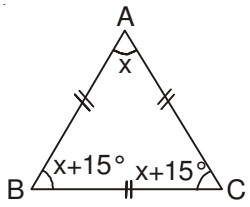
$\therefore$ But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\Rightarrow \quad x + x + 15° + x + 15° = 180°$

$\Rightarrow \quad 3x + 30° = 180°$

$\Rightarrow \quad 3x = 180° - 30° = 150°$

$\Rightarrow \quad x = \frac{150°}{3} = 50°$



$\therefore \quad \angle A = 50°, \angle B = 50° + 15° = 65°$ and $\angle C = 65°$

8. The vertical angle of an isosceles triangle is twice the sum of its base angles. Find each angle of the triangle.

Ans. In an isosceles triangle ABC, AB = AC

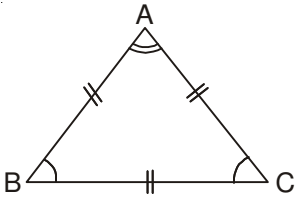
Let each base angle $\angle B = \angle C = x$, then $\angle A = 2x$

But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\therefore \quad 2x + x + x = 180°$

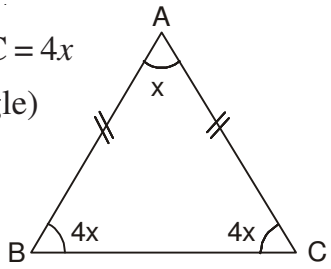
$\Rightarrow \quad 4x = 180° \Rightarrow x = \frac{180°}{4} = 45°$

$\therefore \quad \angle A = 2x = 2 \times 45° = 90° \quad \angle B = 45° \quad \angle C = 45°$



9. In an isosceles triangle, triangle, each base angle is four times its vertical angle. Find each angle of the triangle.

Ans. In an isosceles ΔABC, $AB = AC$



Let vertical angle $\angle A = x$ then each base angle $\angle B = \angle C = 4x$

But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\therefore \quad x + 4x + 4x = 180°$

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = \frac{180°}{9} = 20°$

$\therefore \quad \angle A = 20°,\ \angle B = 4x = 4 \times 20° = 80°,\ \angle C = 4x = 4 \times 20° = 80°$

10. The ratio between the base angle and the vertical angle of an isosceles triangle is 2 : 5. Find each angle of the triangle.

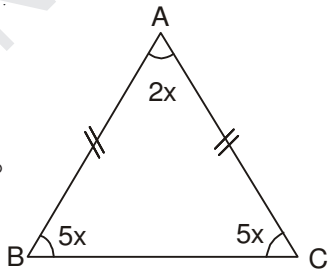
Ans. In an isosceles ΔABC. $AB = AC$

Ratio between vertical angle A and base angle B = 2 : 5

Let vertical angle $A = 2x$ then $\angle B = 5x$ and $\angle C = 5x$

But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\therefore \quad 2x + 5x + 5x = 180°$

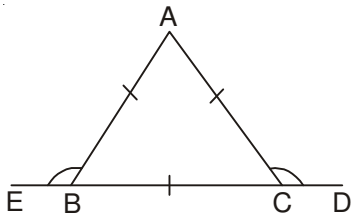


$\Rightarrow \quad 12x = 180° \Rightarrow x = \frac{180°}{12} = 15°$

$\therefore \quad \angle A = 2x = 2 \times 15° = 30°,\ \angle B = 5x = 5 \times 15° = 75°$

$\angle C = 5x = 5 \times 15° = 75°$

11. In the given figure, ΔABC is an equilateral triangle whose side BC has been produced in both the directions to D and E respectively. Prove that $\angle ACD = \angle ABE$.



Ans. In ΔABC, $AB = BC = CA$

$\therefore \quad \angle A = \angle B = \angle C = 60°$

Now ext $\angle ABE = \angle BAC +$

$\angle BCA = 60° + 60° = 120°$ …(*i*)

Similarly ext. $ACD = \angle ABC + \angle BAC = 60° + 60° = 120°$ …(*ii*)

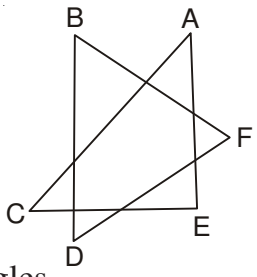
from (*i*) and (*ii*)

$\angle ABE = \angle ACD$ or $\angle ACD = \angle ABE$

12. In the figure given along side, find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$.

Ans. In ΔACE

$\angle A + \angle C + \angle E = 180°$ (Sum of angles of triangle)



In ΔBDF $\angle B + \angle D + \angle F = 180°$ (Sum of angles of triangle)

On adding (*i*) and (*ii*) we have

$\angle A + \angle C + \angle E + \angle B + \angle D + \angle F = 180° + 180°$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

13. If the angles of a triangle are in the ratio 3 : 4 : 11, find the angles.

Ans. Let the angle of a triangle be $3x$, $4x$ and $11x$.

As sum of angles of a triangle is 180°

$\therefore \quad 3x + 4x + 11x = 80°$

$$18x = 180°, \; x = \frac{180°}{18} = 10°$$

$\therefore$ Angles of a triangle are $3x = 3 \times 10° = 30°$

$4x = 4 \times 10° = 40°$ and $11x = 11 \times 10° = 110°$

14. If the two acute angles of a right angled triangle are in the ratio 7 : 8, find these angles.

Ans. Let the two acute angles of a right angled triangle be $7x$, $8x$

Now, $7x + 8x + 90° = 180°$ (Sum of angle of a triangle)

$15x + 90° = 180°$

$15x = 180° - 90°$

$15x = 90°$

$$\Rightarrow \quad x = \frac{90}{15} = 6.$$

$\therefore$ Two acute angles of a right angled triangle are

$7x = 7 \times 6 = 42°$ and $8x = 8 \times 6 = 48°$

15. One angle of a triangle is 66° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{2}{3}$

Find these two angles.

Ans. One angle = 66°. Let the other two angles of a triangle be $\frac{3}{2}x$ and $\frac{5}{3}x$.

Now, sum of angles of a triangle is 180°

$$\therefore \quad 66° + \frac{3}{2}x + \frac{5}{3}x = 180°$$

$$\Rightarrow \quad 66° + \frac{9x + 10x}{6} = 180° \;\Rightarrow\; 66° + \frac{19x}{6} = 180°$$

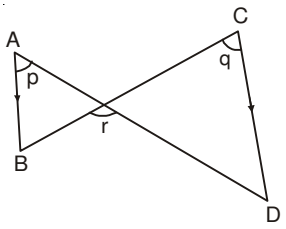
$\Rightarrow \quad \frac{19x}{6} = 180° - 66° = 114°$

$\therefore \quad x = \frac{114° \times 6}{19} = 36°$

$\therefore$ Other two angles are

$\frac{3}{2}x = \frac{3}{2} \times 36° = 54°$ and $\frac{5}{3}x = \frac{5}{3} \times 36° = 60°$

16. In the adjoining figure, AB || CD. Prove that $p + q = r$.



Ans. AB || CD and AD is a transversal

$\therefore \quad \angle ADC = p$ (Alternate angles)

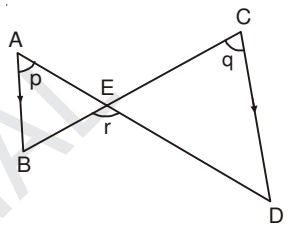
In ΔCED,

$\angle r$ is an exterior angle and angles q and $\angle EDC$ are its opposite interior angles.

$\because$ Exterior angle = sum of its opposite interior angles

$\therefore \quad r = q + \angle EDC = q + p \ [\angle EDC = \angle ADC = p]$

Hence proved.



17. In a ΔABC, $3\angle A = 4\angle B = 6\angle C$. Find $\angle A$, $\angle B$ and $\angle C$.

Ans. Let $3\angle A = 4\angle B = 6\angle C = x°$

$\therefore \quad \angle A = \left(\frac{x}{3}\right)° \quad \angle B = \left(\frac{x}{4}\right)° \quad \angle C = \left(\frac{x}{6}\right)°$

Since, sum of angles of a triangle is 180°

$\therefore \quad \frac{x}{3} + \frac{x}{4} + \frac{x}{6} = 180° \Rightarrow \frac{4x + 3x + 2x}{12} = 180°$

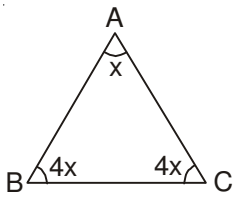
$\therefore \quad \frac{9x}{12} = 180°$

$x = \frac{180 \times 12}{9} = 240°$

$\therefore \quad \angle A = \left(\frac{x}{3}\right)° = \frac{240°}{3} = 80° \quad \angle B = \left(\frac{x}{4}\right)° = \frac{240°}{4} = 60°$

$\angle C = \left(\frac{x}{6}\right)° = \frac{240}{6} = 40°$

18. In an isosceles triangle, a base angle is four times its vertical angle. Find all the angles of the triangle.



Ans. Let the vertical angle be $x°$, then

$$\text{base angle} = 4x$$

In an isosceles Δ, base angles are equal

$$\therefore \quad \angle A = x°,\ \angle B = 4x° \text{ and } \angle C = 4x°$$

Sum of angles of a triangle is 180°

$$\therefore \quad x + 4x + 4x = 180° \quad x = \frac{180°}{9} = 20°$$

$$\therefore \quad \text{Vertical angle} = 20° \text{ and each base angle} = 4 \times 20° = 80°$$

19. In a ΔPQR if $\angle P = 90°$ and $\angle Q = \angle R$, find the measure of each of the equal angles of the triangle.

Ans. Let the measure of each equal angle be $x°$ *i.e.* $\angle Q = \angle R = x°$

We know that in ΔPQR

$$\angle P + \angle Q + \angle R = 180°$$

$$\Rightarrow \quad 90° + x + x = 180° \quad \Rightarrow \quad 90° + 2x = 180°$$

$$\Rightarrow \quad 2x = 180° - 90° \quad \Rightarrow \quad 2x = 90°$$

$$\therefore \quad x = \frac{90°}{2} = 45°$$

Hence, measure of each equal angle is 45° $\angle Q = \angle R = 45°$.

20. In a right-angled triangle, the two acute angles are in the ratio 4 : 5. Find these angles.

Ans. Let the two acute angles are $4x$ and $5x$. In right angled triangle one angle is 90°

$$\therefore \quad 4x + 5x + 90° = 180$$

$$\Rightarrow \quad 9x + 90° = 180° \quad \Rightarrow \quad 9x = 180° - 90°$$

$$\Rightarrow \quad 9x = 90°,\ x = 10°$$

$$\therefore \quad \text{Two acute angles are } 4(10) \text{ and } 5(10) \text{ i.e. } 40° \text{ and } 50°.$$

21. One of the two equal angles of an isosceles triangle measure 65°. Find the measure of each angle of the triangle.

Ans. Let the unequal or third angle be $x°$.

In isosceles triangle $x + 65° + 65° = 180°$

$$\Rightarrow \quad x + 130° = 180°$$

$$\Rightarrow \quad x = 180° - 130° = 50°$$

Hence, measure of each angle of the triangle are 65°, 65° and 50°.

22. In a right angled triangle if one of the acute angles measures 40°, find the measure of the other acute angle.

Ans. Let the measure of the other acute angle be $x°$.

In right angled triangle one angle is 90°

$\Rightarrow \quad x + 90° + 40° = 180°$

$\Rightarrow \quad x + 130° = 180° \Rightarrow x = 180° - 130°$

$\therefore \quad x = 50°$

Hence, the measure of other acute angle is 50°

23. The vertical angle of an isosceles triangle is three times the sum of its base angles. Find all angles of the triangle

Ans. Let each base angle be x

$\therefore$ Vertical angle $= 3\,(x + x) = 3\,(2x) = 6x$

We know that,

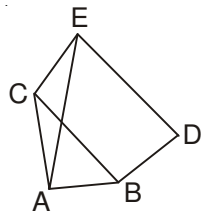
Sum of angles of a triangle is 180°

$\therefore \quad x + x + 6x = 180° \Rightarrow 8x = 180°$

$\therefore \quad x = \dfrac{180°}{8} = 22.5°$

Hence, each base angle = 22.5° and vertical angle $= 6x = 6 \times 22.5° = 135°$

24. In the adjoining figure, ABC is an equilateral triangle and BDEC is a square. Find $\angle AED$.



Ans. $\because \quad \Delta ABC$ is an equilateral triangle.

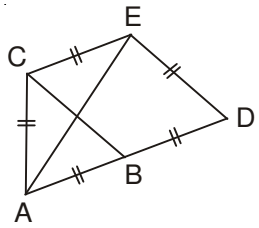
$\therefore \quad$ Each angle of ΔABC is 60°.

$\therefore \quad \angle ACB = 60°$

In ACE, AC = CE

$\therefore \quad \Delta ACE$ is an isosceles triangle

$\therefore \quad \angle CEA = \angle CAE$



Now, vertical angle of ΔACE is $\angle ACE = \angle ACB + \angle ECB = 60° + 90° = 150°$

Now, $\angle ACE + \angle CEA + \angle CAE = 180°$ ($\because$ Sum of angles of triangle is 180°)

$\Rightarrow 150° + \angle CEA + \angle CEA = 180°$ ($\because \angle CEA = \angle CAE$)

$\Rightarrow \quad 2\angle CEA = 180° - 150°$

$\Rightarrow \quad \angle CEA = \dfrac{30°}{2} = 15°$

$\angle CEA = \angle CAE = 15°$

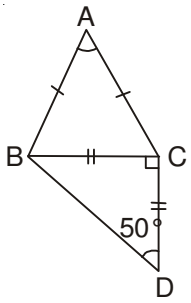
$\angle CED = 90°$ (Angle of a square)

$\Rightarrow \quad \angle CEA + \angle AED = 90°$

$15° + \angle AED = 90°$

$\therefore \quad \angle AED = 90° - 15° = 75°$

25. From the adjoining figure, find $\angle BDC$ and $\angle ABD$.



Ans. In Δ BCD, since BC = CD (Given)

$\therefore \quad \angle BDC = \angle DBC$ (angles opposite to equal sides)

$\angle BDC + \angle DBC + \angle BCD = 180°$ (Sum of angles of a triangle)

$\therefore \quad \angle BDC + \angle BDC + 90° = 180°$

$\Rightarrow \quad 2\angle BDC = 180° - 90°$

$\therefore \quad \angle BDC = \frac{90°}{2} = 45°$

In Δ ABC, AB = AC (Given)

$\therefore \quad \angle ACB = \angle ABC$ (angles opposite to equal sides)

Now, $\angle BAC + \angle ABC + \angle ACB = 180°$ (Sum of angles of a triangle)

$50° + \angle ABC + \angle ABC = 180°$ ($\therefore \angle ABC = \angle ACB$)

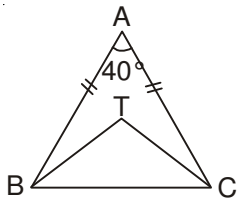
$\therefore \quad 2\angle ABC = 180° - 50°$

$\Rightarrow \quad 2\angle ABC = 130°$

$\Rightarrow \quad \angle ABC = \frac{130°}{2} = 65°$

$\therefore \quad \angle ABD = \angle ABC + \angle CBD = 65° + 45° = 110°$

26. In the given figure, BI is the bisector of $\angle$ABC and CI is the bisector of $\angle$ACB. Find $\angle$BIC.



Ans. In Δ ABC,

BI is the bisector of $\angle$ABC and CI is the bisector of $\angle$ACB.

$\therefore \quad$ AB = AC

$\angle B = \angle C$ (Angles opposite to equal sides)

But $\angle A = 40°$

and $\angle A + \angle B + \angle C = 180°$ (Angles of a triangle)

$\Rightarrow \quad 40° + \angle B + \angle B = 180°$

$\Rightarrow \quad 40° + 2\angle B = 180°$

$\Rightarrow \quad 2\angle B = 180 - 40° = 140°$

$\Rightarrow \quad \angle B = \frac{140°}{2} = 70°$

$\therefore \quad \angle ABC = \angle ACB = 70°$

But BI and CI are the bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$\therefore \quad \angle IBC = \frac{1}{2}\angle ABC = \frac{1}{2}(70°) = 35°$

and $\quad \angle ICB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 70° = 35°$

Now, in ΔIBC,

$\angle BIC + \angle IBC + \angle ICB = 180°$ (Angles of a triange)

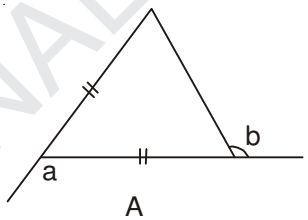
$\Rightarrow \quad \angle BIC + 35° + 35° = 180°$

$\Rightarrow \quad \angle BIC + 70° = 180°$

$\Rightarrow \quad \angle BIC = 180° - 70° = 110°$

Hence, $\angle BIC = 110°$

27. In the given figure express a in terms of b.



Ans. In ΔABC,

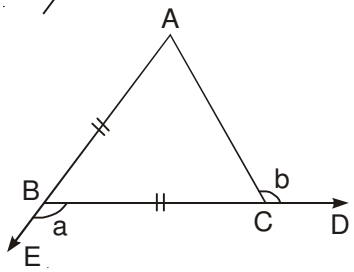
$BC = BA$

$\therefore \quad \angle BCA = \angle BAC \quad ...(i)$

and exterior $\quad \angle CBE = \angle BCA + \angle BAC$

$\Rightarrow \quad a = \angle BCA + \angle BCA \quad$ [by (i)]

$\Rightarrow \quad a = 2\ \angle BCA$



But $\angle ACB = 180° - b$ ($\because \angle ACD$ and $\angle ACB$ are linear pair)

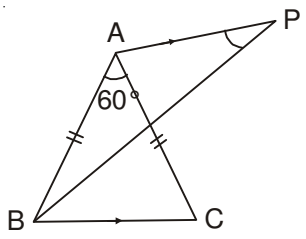
$\Rightarrow \quad \angle BCA = 180° - b$

$\because \quad a = 2\ \angle BCA = 2\ (180° - b) = 360° - 2b$

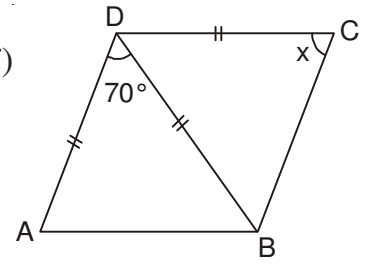
28. Find x in Figure

Given: DA = DB = DC, BD bisects $\angle ABC$ and $\angle ADB = 70°$.

(i)



(ii)



Ans. In the figure,

$$DA = DB = DC$$

BD bisects $\angle ABC$ and $\angle ADB = 70°$

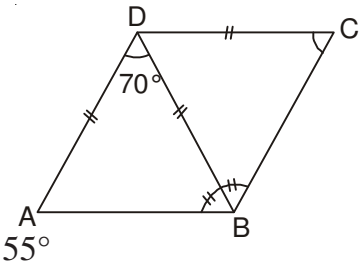
But $\angle ADB + \angle DAB + \angle DBA = 180°$ (Angle of a triangle)

$\Rightarrow$ $70° + \angle DBA + \angle DBA = 180°$ ($\because$ DA = DB)

$\Rightarrow$ $70° + 2\angle DBA = 180°$

$\Rightarrow$ $2\angle DBA = 180° - 70° = 110°$

$\therefore$ $\angle DBA = \frac{110°}{2} = 55°$



$\because$ BD is the bisector of $\angle ABC$, $\angle DBA = \angle DBC = 55°$

But in ΔDBC,

$$DB = DC$$

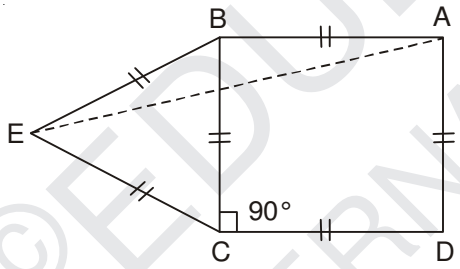
$\therefore$ $\angle DCB = \angle DBC$

$\Rightarrow$ $x = 55°$

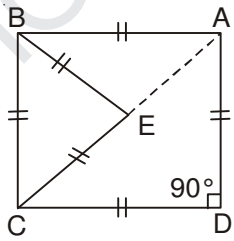
29. In each figure, given below, ABCD is a square and ΔBEC is an equilateral triangle. Find in each case :

(*i*) $\angle ABE$ (*ii*) $\angle BAE$

(*i*)



(*ii*)



Ans. We know that the sides of a square are equal and each angle is of 90°

Three sides of an equilateral triangle are equal and each angle is of 60°. Therefore in fig. (*i*), ABCD is a square and ΔBEC is an equilateral triangle.

(*i*) $\angle ABE = \angle ABC + \angle CBE = 90° + 60° = 150°$

(*ii*) But in ΔABE

$\angle ABE + \angle BEA + \angle BAE = 180°$ (Angles of a triangle)

$\Rightarrow 150° + \angle BAE + \angle BAE = 180°$ ($\because$ AB = BE)

$\Rightarrow$ $150° + 2\angle BAE = 180°$

$\Rightarrow$ $2\angle BAE = 180° - 150° = 30°$

$\therefore$ $\angle BAE = \frac{30°}{2} = 15°$

In figure (*ii*),

$\because$ ABCD is square and ΔBEC is an equilateral triangle,

$\therefore$ $\angle ABE = \angle ABC - \angle CBE$

$= 90° - 60° = 30°$

In ΔABE, $\angle ABE$ $\angle AEB + \angle BAE = 180°$ (Angles of a triangle),

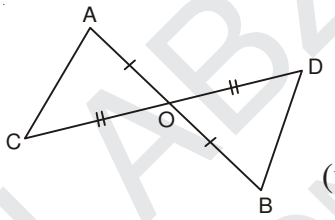
$\Rightarrow$ $30° + \angle BAE + \angle BAE = 180°$ [AB = BE]

$\Rightarrow$ $\angle BAE = 180° - 30° = 150°$

$\Rightarrow$ $\angle BAE = \frac{150°}{2} = 75°$

30. In the given figure, we have AO = BO and CO = DO

Prove that : (*i*) $\Delta AOC = \Delta BOD$ (*ii*) AC = BD



Ans. (*i*) In ΔAOC an ΔBOD

AO = BO (given)

CO = DO (given)

$\angle AOC = \angle BOD$ (vertically opposite angles)

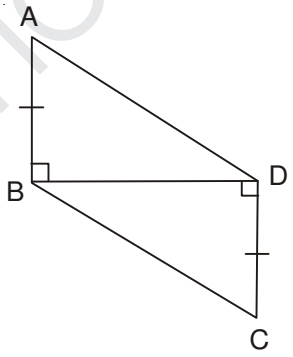
$\Delta AOC \cong \Delta BOD$ (S.A.S)

(*ii*) AC = BD (C.P.C.T)

31. In the given figure, AB $\perp$ BD and AB = CD.

Prove that :

(*i*) $\Delta ABD \cong \Delta CDB$ (*ii*) AD = CB.



Ans. (*i*) In ΔABD and ΔCDB

AB = CD (given)

BD = BD (common)

$\angle ABD = \angle CDB$ (each 90°)

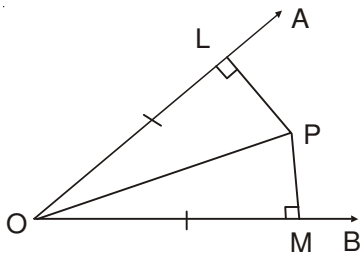
$\therefore$ $\Delta ABD \cong \Delta CDB$ (S.A.S)

(*ii*) AD = CB (C.P.C.T)

32. In the given figure, PL $\perp$ OA and PM $\perp$ OB such that OL = OM. Prove that.

(*i*) $\Delta OLP \cong \Delta OMP$

(*ii*) PL = PM (*iii*) $\angle LOP = \angle MOP$



Ans. In the figure, $\angle L = \angle M = 90°$, OL = OM

Now, in two right triangles

ΔOLP and ΔOMP

OP = OP (common)

OL = OM (given)

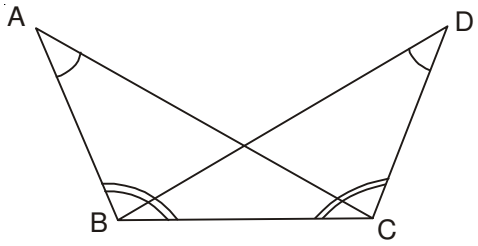
$\angle L = \angle M = 90°$

$\therefore$ $\Delta OLP \cong \Delta OMP$ (by R.H.S. axiom)

(*ii*) $\therefore$ PL = PM (*c.p.c.t*)

(*iii*) and $\angle LOP = \angle MOP$ (*c.p.c.t*)

33. In the adjoining diagram,



$\angle BAC = \angle BDC$ and

$\angle ACB = \angle DBC$.

Prove that AC = BO

Ans. In ΔABC and ΔBDC

$\angle BAC = \angle BDC$ (Given)

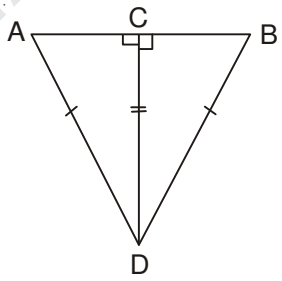
$\angle ACB = \angle DBC$ (Given)

BC = BC (Common)

$\therefore$ $\Delta ABC \cong \Delta DCB$ (By A.A.S axiom of congruency)

$\therefore$ AC = BD (Corresponding parts of congruent triangles)

34. In the given figure, we have AC $\perp$ CD, BC $\perp$ CD and DA = DB. Prove that CA = CB.



Ans. In ΔACD and ΔBCD

AD = BD (Given)

$\angle ACD = \angle BCD$ (each 90°)

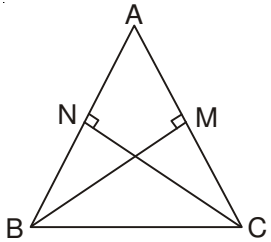
CD = CD (common)

$\therefore$ $\Delta ACD \cong \Delta BCD$ (S.A.S.)

$\therefore$ CA = CB (C.P.C.T)

35. In the adjoining figure, ΔABC is an isosceles triangle in which AB = AC. If BM $\perp$ AC and CN $\perp$ AB Prove that :

(*i*) $\Delta BMC \cong \Delta CNB$ (*ii*) BM = CN



Ans. In ΔABC, AB = AC

BM $\perp$ AC and CM $\perp$ AB

To prove : (*i*) $\Delta BMC \cong \Delta CNB$

(*ii*) BM = CN

Proof : $\because$ AB = AC

$\therefore$ $\angle ACB = \angle ABC$ (angles opposite to equal sides)

Now in ΔBMC and ΔCNB

BC = BC (common)

$\angle M = \angle N$ (each 90°)

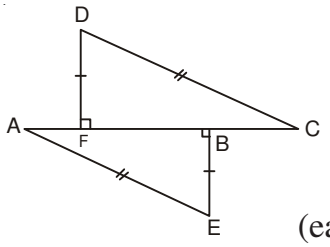
$\angle ACB = \angle ABC$ [$\because$ AB = AC]

$\therefore$ $\Delta BMC \cong \Delta CNB$ (AAS axiom)

$\therefore$ BM = CN (CPCT.)

36. In the figure given alongside prove that

(*i*) AB = FC (*ii*) AF = BC.



Ans. In ΔABE and ΔDFC

$\angle B = \angle F$ (each 90°)

AE = DC (Given)

BE = DF (Given)

$\therefore$ $\Delta ABE \cong \Delta CFD$

(R.H.S congruence rule)

(*i*) $\therefore$ AB = FC (Corresponding parts of congruent triangles)

(*ii*) As AB = FC (Proved above)

$\Rightarrow$ AF + FB = FB + BC

$\Rightarrow$ AF + FB – FB = BC

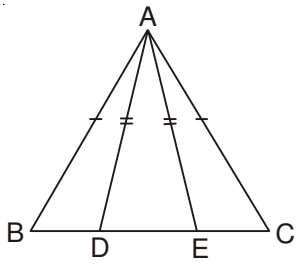
$\Rightarrow$ AF = BC

37. In the adjoining figure, AB = AC and AD = AE.

Prove that (*i*) $\angle ADB = \angle AEC$

(*ii*) $\Delta ABD \cong \Delta ACE$

(*iii*) BE = DC.



Ans. In the given figure

AD = AE (Given)

$\therefore$ $\angle ADE = \angle AED$ (Corresponding angles opposite to equal sides)

Now, $\angle ADE + \angle ADB = \angle AED + \angle AEC$ (Linear pair angles)

As $\angle ADE = \angle AED$

$\therefore$ $\angle ADB = \angle AEC$

(*ii*) In ΔABD and ΔAEC

AB = AC so, $\angle B = \angle C$

AD = AE

$\angle ADB = \angle AEC$ then, $\angle BAD = \angle EAC$ (Prove above)

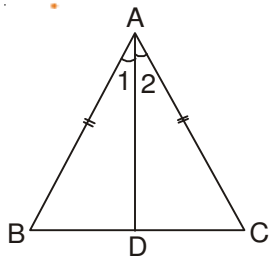
$\therefore$ $\Delta ABD \cong \Delta ACE$ (SAS congruence rule)

(*iii*) Since $\Delta ABD \cong \Delta ACE$

$BD = EC$ (by CPCT)

$\Rightarrow$ $BD + DE = EC + DE$

$\Rightarrow$ $BE = CD$



38. In the given figure; $\angle 1 = \angle 2$ and $AB = AC$. Prove that:

(*i*) $\angle B = \angle C$ (*ii*) $BD = DC$

(*iii*) AD is perpendicular to BC.

Ans. Proof :

In ΔADB and ΔADC,

$AB = AC$ (given)

$\angle 1 = \angle 2$ (given)

$AD = AD$ (common)

$\therefore$ $\Delta ADB \cong \Delta ADC$ (S.A.S. Axiom)

(*i*) Hence $\angle B = \angle C$ (CPCT)

(*ii*) $BD = DC$ (CPCT)

(*iii*) $\angle ADB = \angle ADC$ (CPCT)

But $\angle ADB + \angle ADC = 180°$ (Linear pair angles)

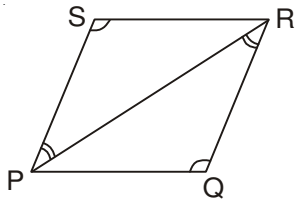
$\therefore$ $\angle ADB = \angle ADC = 90°$

Hence AD is perpendicular to BC.

39. In the given figure prove that :

(*i*) $PQ = RS$ (*ii*) $PS = QR$

Ans. Proof : In ΔPQR and ΔPSR,



$PR = PR$ (common)

$\angle PRQ = \angle RPS$ (given)

$\angle PQR = \angle PSR$ (given)

$\therefore$ $\Delta PQR \cong \Delta RSP$ (A.A.S..Axiom)

$PQ = RS$ (CPCT)

(*ii*) $QR = PS$ (CPCT)

or $PS = QR$

40. Construct a triangle ABC such that $AB = BC = 4.7$ cm and $\angle B = 75°$. Measure angles A and C.

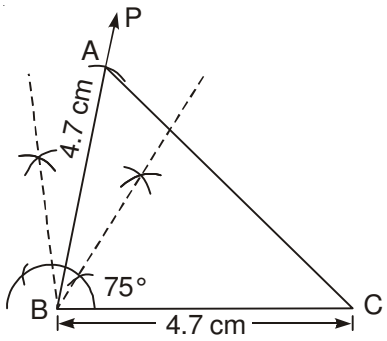
Ans. Steps of construction:

1. Draw a line segment $BC = 4.7$ cm.

2. At B, construct $\angle CBP = 75°$.

3. With B as centre and radius 4.7 cm (= BA) draw an arc to meet BP at A.

4. Join AC, then ABC is the required triangle $\angle A = 52.5°$ and $\angle C = 52.5°$.



41. Construct a triangle PQR given that PQ = 4.9 cm, $\angle P = 45°$ and $\angle Q = 60°$ Measure $\angle R$ from R, draw a perpendicular to PQ.

Ans. Steps of construction :

1. Draw a line segment PQ = 4.9 cm

2. At P, construct $\angle QPM = 45°$.

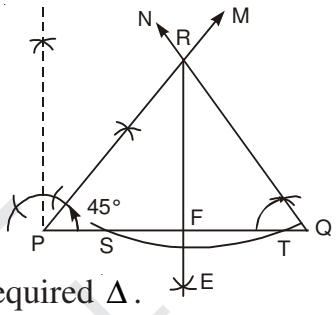
3. At Q construct $\angle PQN = 60°$.

4. Let PM and QN intersect at R then PQR is the the required Δ.
On measuring $\angle R = 75°$

5. With R as centre and any suitable radius, draw an arc to cut the line PQ at points S and T.

6. With S and T as centres and radius more than $\frac{1}{2}$ ST, draw two arcs interescting each other at E on the other side of PQ.

7. Join R and E to meet the line PQ at F, then RF is the required perpendicular to the line PQ.



42. Construct an isosceles triangle PQR with base PQ = 4.3 cm and $\angle Q = 75°$. Measure $\angle R$.

Ans. Since the base angles of an isosceles triangle are equal, therefore, $\angle P = \angle Q = 75°$.

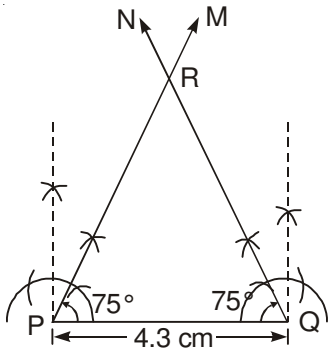
Steps of construction:

1. Draw PQ = 4.3 cm.

2. At P, construct $\angle QPM = 75°$.

3. At Q construct $\angle PQN = 75°$.

4. Let PM and QN intersect at R, then PQR is the required Δ. On measuring $\angle R = 30°$.



43. Construct a right angled triangle ABC such that BC = 3.6 cm, $\angle B = 90°$ and $\angle A = 60°$.

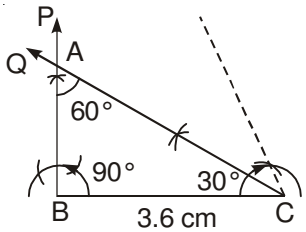
Ans. We know that. Sum of angles of a triangle is 180°

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 60° + 90° + \angle C = 180°$

$\Rightarrow \quad 150° + \angle C = 180°$

$\therefore \quad \angle C = 180° - 150° = 30°$



Steps of construction:

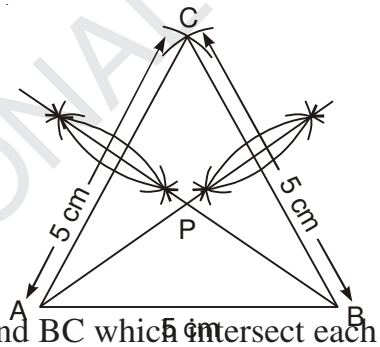
1. Draw BC = 3.6 cm.
2. At B, construct $\angle CBP = 90°$.
3. At C, construct $\angle BCQ = 30°$
4. Let BP and CQ intersect at A. Then, ABC is the required right angled triangle.

44. Construct an equilateral Δ ABC such that

(*i*) AB = 5 cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.

(*ii*) Each side is 6cm.

Ans. (*i*) Steps of Construction :



(*i*) Draw a line segment AB = 5 cm.

(*ii*) With A and B as centre and radius 5 cm each, draw two arcs intersecting each other at C.

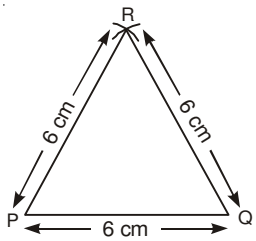
(*iii*) Join AC and BC

then, Δ ABC is the required triangle.

(*iv*) Draw the perpendicular bisectors of sides AC and BC which intersect each other at P.

(*v*) Join AP,PB and PC. On measuring, each is 2.8 cm.

(*ii*) Steps of Construction :



(*i*) We draw a line segment PQ = 6 cm.

(*ii*) With P and Q as centre and 6 cm as radius draw two arcs intersecting each other at C.

(*iii*) Join PR and QR

Thus ΔPQR is the required triangle.

45. (*i*) Construct a Δ ABC such that AB = 4 cm, BC = 2.6 cm and CA = 3.5cm. Inscribe a circle to this triangle and measure its radius.

(*ii*) Construct an isosceles Δ MNP such that base MN = 5 cm, base angle MNP = 30°. Construct an incircle to this triangle and measure its radius.

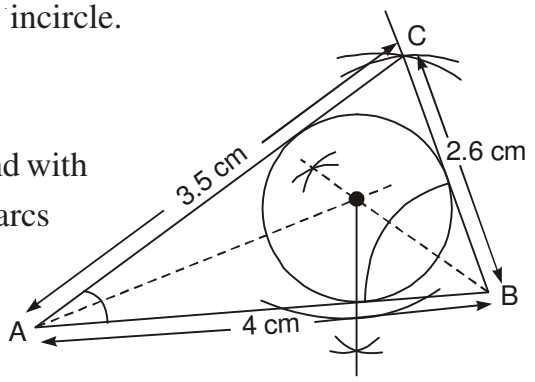
(*iii*) Construct an equilateral Δ DEF whose one side is 5.5 cm. Construct an incircle to this triangle.

(*iv*) Construct a ΔPQR such that PQ = 6 cm, $\angle QPR = 45°$, and $\angle PQR = 60°$. Locate its incentre and then draw its incircle.

Ans. (*i*) Steps of Construction :

(*i*) Draw a line segment AB = 4 cm.

(*ii*) With A as centre and radius 3.5 cm and with B as centre and radius 2.6 cm, draw arcs intersecting each other at C.



(*iii*) Join AC and BC.

(*iv*) Draw the angle bisectors of $\angle A$ and $\angle B$ intersecting each other at I.

(*v*) From I draw IL $\perp$ AB

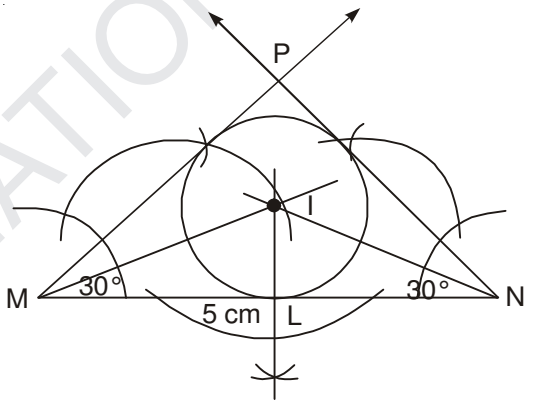
(*iv*) With centre I and radius IL, draw a circle which touches the sides of ΔABC internally. This is the required incircle.

On measuring the required radius of this incircle is 1cm.

(*ii*) Steps of Construction:

(*i*) We draw a line segment MN = 5 cm.

(*ii*) At M and N, draw two rays making an angle of 30° each which intersect each other at P.



(*iii*) Now draw the angle bisectors of $\angle M$ and $\angle N$ which intersect each other at I.

(*iv*) From I , draw perpendicular IL on MN.

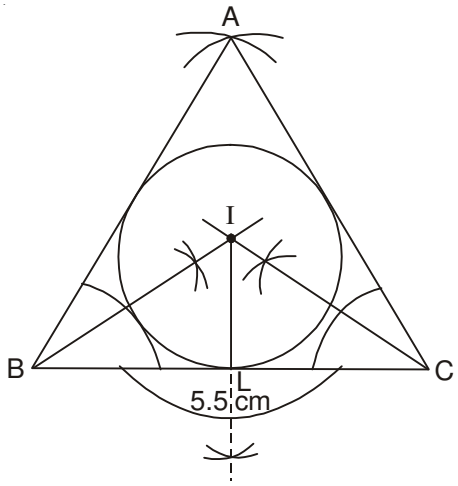
(*v*) With centre I and radius IL, draw a circle which touches the sides of the ΔPMN internally.

On measuring the required incircle and its radius is 0.7 cm.

(*iii*) Steps of Construction :

(*i*) Draw a line segment BC = 5.5 cm

(*ii*) With B and C as centre C and radius 5.5 cm each draw two arcs intersecting each other at A.



(*iii*) Join AB and AC.

(*iv*) Draw the perpendicular bisectors of $\angle B$ and $\angle C$ interecting each other at I.

(*v*) From I, draw IL ⊥ BC

(*vi*) With centre I and radius IL, draw a circle which touches the sides of the Δ ABC internally.

This is the required incircle.

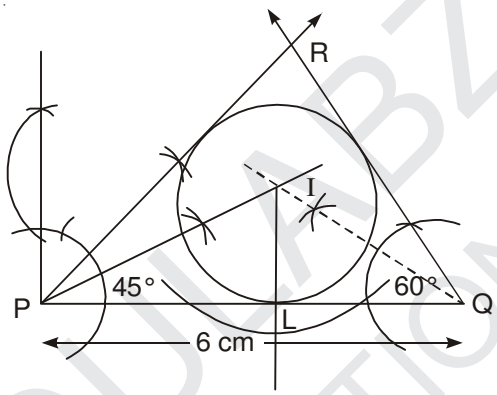
(*iv*) Steps of Construction:

(*i*) Draw a line segment PQ = 6 cm.

(*ii*) At P draw a ray making an angle of 45° and at Q draw a ray making an angle of 60° intersecting each other at R.

(*iii*) Draw the bisectors of ∠P and ∠Q intersecting each other at I.

(*iv*) From I, draw IL ⊥ PQ.



(*v*) With centre I and radius IL, draw a circle which touches the sides of a Δ PQR internally. This is the required incircle whose I is incentre.