15 FACTORS AND MULTIPLES

1.(A) Using test of divisibility, find which of the following numbers are divisible by 3 :

(*i*) 3624 (*ii*) 81756 (*iii*) 142367

(*iv*) 672588 (*v*) 105756 (*vi*) 256784

Ans. (*i*) Sum of digits of given number = 3 + 6 + 2 + 4 = 15, which is also divisible by 3.

$\therefore$ 3624 is divisible by 3.

(*ii*) Sum of digits of given number = 8 + 1 + 7 + 5 + 6 = 27, which is divisible by 3.

$\therefore$ 81756 is also divisible by 3.

(*iii*) Sum of digits of given number = 1 + 4 + 2 + 3 + 6 + 7 = 23, which is not divisible by 3.

$\therefore$ 142367 is not divisible by 3.

(*iv*) Sum of digits of given number = 6 + 7 + 2 + 5 + 8 + 8 = 36, which is divisible by 3.

$\therefore$ 672588 is divisible by 3.

(*v*) Sum of digits of given number = 1 + 0 + 5 + 7 + 5 + 6 = 24, which is divisible by 3.

$\therefore$ 105756 is also divisible by 3.

(*vi*) Sum of digits of given number = 2 + 5 + 6 + 7 + 8 + 4 = 32, which is not divisible by 3.

$\therefore$ 256784 is not divisible by 3.

(B) Which of the following numbers are divisible by 4 ?

(*i*) 14584 (*ii*) 30218 (*iii*) 24560

(*iv*) 53876 (*v*) 19594 (*vi*) 36912

Ans. (*i*) Last two digits of given numbr = 84, which is divisible by 4

$\therefore$ 14584 is divisible by 4.

(*ii*) Last two digits of given number = 18, which is not divisible by 4

$\therefore$ 30218 is not divisible by 4.

(*iii*) Last two digits of given number = 60, which is divisible by 4

$\therefore$ 24560 is divisible by 4.

(*iv*) Last two digits of given number = 76, which is divisible by 4

$\therefore$ 53876 is divisible by 4.

(*v*) Last two digits of given number = 94, which is not divisible by 4.

$\therefore$ 19594 is not divisible by 4.

(*vi*) Last two digits of given number = 12, which is divisible by 4.

$\therefore$ 36912 is divisible by 4.

(C) Which of the following number are divisible by 11?

(*i*) 357269 (*ii*) 110111 (*iii*) 346929

(*iv*) 517633 (*v*) 245795 (*vi*) 95827

Ans. (*i*) (Sum of digits at odd places) – (Sum of digits at even places)

= (9 + 2 + 5) – (6 + 7 + 3) = 16 – 16 = 0

$\therefore$ 357269 is divisible by 11.

(*ii*) (1 + 1+ 1) – (1 + 0 + 1) = 3 – 2 = 1

$\therefore$ 110111 is not divisible by 11.

(*iii*) (9 + 9 + 4) – (2 + 6 + 3) = 22 – 11 = 11.

$\therefore$ 346929 is divisible by 11.

(*iv*) (3 + 6 + 1) – (3 + 7 + 5) = 10 – 15 = –5

$\therefore$ 517633 is not divisible by 11.

(*v*) (5 + 7 + 4) – (9 + 5 + 2) = 16 – 16 = 0

$\therefore$ 245795 is divisible by 11.

(*vi*) (7 + 8 + 9) – (2 + 5) = 24 – 7 = 17

$\therefore$ 95827 is not divisible by 11.

(D) Which of the following numbers are divisible by 6.

(*i*) 97234 (*ii*) 16524 (*iii*) 3825 (*iv*) 52618

Ans. (*i*) Last digit of 97234 is 4, which is divisible by 2.

Now, sum of its digits = 9 + 7 + 2 + 3 + 4 = 25, which is not divisible by 3.

$\therefore$ 97234 is not divisible by 6.

(*ii*) Last digit of 16524 is 4, which is divisible by 2.

Now, sum of its digits = 1 + 6 + 5 + 2 + 4 = 18, which is divisible by 3.

$\therefore$ 16524 is divisible by 6.

(*iii*) Last digit of 3825 is 5, which is not divisible by 2.

$\therefore$ 3825 is not divisible by 6.

(*iv*) Last digit of 52618 is 8, which is divisible by 2.

Now, sum of is digits = 5 + 2 + 6 + 1 + 8 = 22, which is not divisible by 3.

$\therefore$ 52618 is not divisible by 6.

2. Find the smallest 6-digit number which is exactly divisible by 239.

Ans. The smallest 6-digit number = 100000

We divide by 100000 by 239.

$$
\begin{array}{r}
239\overline{)100000}(418 \\
\underline{956} \\
440 \\
\underline{239} \\
2010 \\
\underline{1912} \\
98
\end{array}
$$

Hence, the least number which should be added to 100000, so that the sum is exactly divisible by 239.

Then, 239 – 98 = 141.

Hence, the smallest digit of whole number which is exactly divisible by 239 = 100000 + 141 = 100141

3.(A) Find the smallest number which must be added to 9373, so that it becomes divisible by 4.

Ans. Divide 9373 by 4, find the remainder

$$
\begin{array}{r}
4\overline{)9373}(2343 \\
\underline{8} \\
13 \\
\underline{12} \\
17 \\
\underline{16} \\
13 \\
\underline{12} \\
1
\end{array}
$$

Hence, the least number which should be added to 9373, so that the sum is exactly divisible by 4

Then, $4 - 1 = 3$.

(B) Find the smallest which must be added to 605329, so that it becomes divisible by 9

Ans. Divide 605329 by 9.

$$
\begin{array}{r}
9\overline{)605329}(67258 \\
\underline{54} \\
65 \\
\underline{63} \\
23 \\
\underline{18} \\
52 \\
\underline{45} \\
79 \\
\underline{72} \\
7
\end{array}
$$

$\therefore$ The least number which should be added to 605329, so that the sum is exactly divisible by 9.

Then, $9 - 7 = 2$.

4.(A) Replace the letter x in the number $8x\,516$ by the smallest digit, so that the number becomes divisible by 6.

Ans. We have notice that a given number $8x\,516$ is an even number for all values of x because there is 6 at its ones place. By divisibility test, the given number will become divisible by 6, if the sum of digits is divisible by 3.

Sum of digits in the given number = $8 + x + 5 + 1 + 6 = 20 + x$.

The next highest number to 20 which is divisible by 3 is 21

$\therefore \quad 20 + x = 21$

$\Rightarrow \quad x = 21 - 20 = 1$

Hence, the smallest value of x is 1.

(B) Replace the letter a in the number 6a712 by a digit, so that the number may become divisible by 6. There may be more than one solution.

Ans. The given number 6a712 is an even number for all values of a because there is 2 at its ones place. By divisibility test, the given number will become divisible by 6, if the sum of digits is divisible by 3.

Sum of digits in the given number = $6 + a + 7 + 1 + 2 = 16 + a$

The next highest number to 16, which is divisible by 3 is 18.

$\therefore \quad 16 + a = 18 \quad \Rightarrow \quad a = 18 - 16 = 2$

Also, for the possible value,

$16 + a = 21 \quad \Rightarrow \quad a = 21 - 16 = 5$

and $16 + a = 24 \quad \Rightarrow \quad a = 24 - 16 = 8$

Hence, the possible value of a are 2, 5, 8.

5. Use prime factorisation method to find the HCF of the following numbers :

(*i*) 1260, 2592 (*ii*) 280, 315, 385

(*iii*) 876, 792, 1512

Ans. (*i*) 1260, 2592

2	1260
2	630
3	315
3	105
5	35
7	7
	1

2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$2592 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Thus, HCF of 1260, 2592 $= 2 \times 2 \times 3 \times 3 = 36$

(*ii*) 280, 315, 385

2	280
2	140
2	70
5	35
7	7
	1

3	315
3	105
5	35
7	7
	1

5	385
7	77
11	11
	1

$\therefore$ $280 = 2 \times 2 \times 2 \times 5 \times 7$

$315 = 3 \times 3 \times 5 \times 7$

$385 = 5 \times 7 \times 11$

Thus, HCF of 280, 315 and 385 $= 5 \times 7 = 35$.

(*iii*) 576, 792, 1512

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

2	792
2	396
2	198
3	99
3	33
11	11
	1

2	1512
2	756
2	378
3	189
3	63
3	21
7	7
	1

$\therefore$ $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$

$1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$

Thus, HCF of 576, 792 and 1512 = 2 × 2 × 2 × 3 × 3 = 72

6. Using long division method find the HCF of the following numbers:

(*i*) 2923, 3239 (*ii*) 204, 1190, 1445

(*iii*) 1701, 2106, 2754

Ans. (*i*) 2923, 3239

$$
\begin{array}{r}
2923\overline{)3239}(1 \\
2923 \\
\hline
316\overline{)2923}(9 \\
2844 \\
\hline
79\overline{)316}(4 \\
316 \\
\hline
\times \\
\hline
\end{array}
$$

Thus, HCF of 2923 and 3239 = 79

(*ii*) 204, 1190, 1445

First find the HCF of 204 and 1190

$$
\begin{array}{r}
204\overline{)1190}(5 \\
1020 \\
\hline
170\overline{)204}(1 \\
170 \\
\hline
34\overline{)170}(5 \\
170 \\
\hline
\times \\
\hline
\end{array}
$$

∴ HCF of 204 and 1190 = 34

HCF of 34 and 1445

$$
\begin{array}{r}
34\overline{)1445}(4 \\
136 \\
\hline
34\overline{)85}(2 \\
68 \\
\hline
17\overline{)34}(2 \\
34 \\
\hline
\times \\
\hline
\end{array}
$$

Thus, HCF of 204, 1190 and 1445 = 17

(*iii*) 1701, 2106, 2754

HCF of 1701 and 2106

$$\begin{array}{r} 1701\overline{)2106}(1 \\ 1701 \\ \hline 405\overline{)1701}(4 \\ 1620 \\ \hline 81\overline{)405}(5 \\ 405 \\ \hline \times \end{array}$$

$\therefore$ HCF of 1701 and 2106 = 81

Now find the HCF of 81 and 2754.

$$\begin{array}{r} 81\overline{)2754}(34 \\ 243 \\ \hline 324 \\ 324 \\ \hline \times \end{array}$$

Thus, HCF of 1701, 2106 and 2754 = 81

7. Find the LCM of the following numbers using prime factorisation method :

(*i*) 9, 12, 15, 18, 24, 56 (*ii*) 28, 36, 42, 54, 60

(*iii*) 1620, 1728, 1890

Ans. (*i*) 9, 12, 15, 18, 24, 56

$9 = 3 \times 3$

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$56 = 2 \times 2 \times 2 \times 7$

Thus, LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

(*ii*) 28, 36, 42, 54, 60

$28 = 2 \times 2 \times 7$

$36 = 2 \times 2 \times 3 \times 3$

$42 = 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

Thus, LCM $= 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

(*iii*) 1620, 1728, 1890

2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

2	1890
3	945
3	315
3	105
5	35
7	7
	1

$1620 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$1890 = 2 \times 3 \times 3 \times 3 \times 5 \times 7$

Thus, LCM $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 7$

$= 181440$

8. Find the LCM of the following number using common division method :

(*i*) 72, 192, 240 (*ii*) 168, 180, 330

(*iii*) 180, 384, 432

Ans. (*i*) 72, 192, 240

2	72, 192, 240
2	36, 96, 120
2	18, 48, 60
2	9, 24, 30
3	9, 12, 15
	3, 4, 5

Thus, LCM of 72, 192, 240

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 \times 5 = 2880$

(*ii*) 168, 180, 330

2	168, 180, 330
2	84, 90, 165
3	42, 45, 165
5	14, 15, 55
	14, 3, 11

Thus, LCM of 168, 180, 330

$= 2 \times 2 \times 3 \times 5 \times 14 \times 3 \times 11 = 27720$

(*iii*) 180, 384, 432

2	180, 384, 432
2	90, 192, 216
2	45, 96, 108
2	45, 48, 54
3	45, 24, 27
3	15, 8, 9
	5, 8, 3

Thus, LCM of 180, 384, 432

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 8 = 17280.$

9. Find the HCF and LCM of the following fractions :

(*i*) $\frac{4}{5}, \frac{3}{10}, \frac{7}{15}$ (*ii*) $\frac{2}{3}, \frac{8}{15}, \frac{20}{27}, \frac{40}{81}$ (*iii*) $\frac{3}{7}, \frac{9}{14}, \frac{18}{35}, \frac{15}{28}$

Ans. (*i*) $\frac{4}{5}, \frac{3}{10}, \frac{7}{15}$

1	4	3	7

5	5	10	15
	1	2	3

$\therefore$ HCF of given fractions $= \frac{\text{HCF of } 4,3,7}{\text{LCM of } 5,10,15} = \frac{1}{30}$.

And, LCM of given fractions $= \frac{\text{LCM of } 4,3,7}{\text{HCF of } 5,10,15} = \frac{84}{5}$.

(*ii*) $\frac{2}{3}, \frac{8}{15}, \frac{20}{27}, \frac{40}{81}$

2	2	8	20	40
2	1	4	10	20
2	1	2	5	10
5	1	1	5	5
	1	1	1	1

3	3	15	27	81
3	1	5	9	27
3	1	5	3	9
	1	5	1	3

$\therefore$ HCF of given fractions $= \frac{\text{HCF of } 2,8,20,40}{\text{LCM of } 3,15,27,81} = \frac{2}{405}$.

And, LCM of given fractions $= \frac{\text{LCM of } 2,8,20,40}{\text{HCF of } 3,15,27,81} = \frac{40}{3}$.

(*iii*) $\frac{3}{7}, \frac{9}{14}, \frac{18}{35}, \frac{15}{28}$

2	3	9	18	15
3	3	9	9	15
3	1	3	3	5
	1	1	1	5

2	7	14	35	28
7	7	7	35	14
	1	1	5	2

Thus, HCF of given fractions $= \frac{\text{HCF of } 3,9,18,15}{\text{LCM of } 7,14,35,28} = \frac{3}{140}$.

And, LCM of given fractions $= \frac{\text{LCM of } 3,9,18,15}{\text{HCF of } 7,14,35,28} = \frac{90}{7}$.

10. Find the least number which on adding 7 is exactly divisible by 15, 35 and 48.

Ans. LCM of 15, 35 and 48

3	15, 35, 48
5	5, 35, 16
	1, 7, 16

LCM = $3 \times 5 \times 7 \times 16 = 1680$

As per as condition, we need a smallest number which on adding 7 is exactly divisible by 15, 35 and 48.

Thus, the required number = 1680 – 7 = 1673.

11. Find the greatest number of four digits which is exactly divisible by each of 12, 18, 40 and 45.

Ans. LCM of 12, 18, 48 and 45

2	12, 18, 40, 45
2	6, 9, 20, 45
3	3, 9, 10, 45
3	1, 3, 10, 15
5	1, 1, 10, 5
	1, 1, 2, 1

Thus LCM of given number = $2 \times 2 \times 3 \times 3 \times 5 \times 2 = 360$

Greatest number of 4-digits = 9999

As per condition, we need a greatest number of four digits which is exactly divisible by 360

$$\begin{array}{r|l} 360 & 9999 \\ & 720 \\ \hline & 2799 \\ & 2520 \\ \hline & 279 \end{array} \quad (27$$

Thus, the required number = 9999 – 279 = 9720.

12. Find the least number of five digits which is exactly divisible by each of 32, 36, 60, 90 and 144.

Ans. LCM of 32, 36, 60, 90 and 144.

2	32,	36,	60,	90,	144
2	16,	18,	30,	45,	72
2	8,	9,	15,	45,	36
2	4,	9,	15,	45,	18
3	2,	9,	15,	45,	9
3	2,	3,	5,	15,	3
5	2,	1,	5,	5,	1
	2,	1,	1,	1,	1

Thus, LCM of given numbers

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 2 = 1440$

Least number of 5 digits = 10000

$$\begin{array}{r|l} 1440 & 10000 \\ & 8640 \\ \hline & 1360 \end{array} \quad (6$$

As per as condition, we need a smallest number of five digits which is exactly divisible by 1440.

Thus, the required number = 10000 + (1440 – 1360)

= 10000 + 80 = 10080.

13. Three drums of capacities 165 litres, 240 litres and 240 litres are to be filled from a tank by using a mug an intergral number of times in each case. What is the largest capacity of the mug ?

Ans. Given three drums of capacities are 165 litres, 195 litres and 240 litres.

For the largest capacity of the mug we will find the HCF of 165 litres 195 litres and 240 litres.

First of all we will find the HCF of 165 litres and 195 litres

$$
\begin{array}{r}
165\overline{)195}(1 \\
165 \\
\hline
30\overline{)165}(5 \\
150 \\
\hline
15\overline{)30}(2 \\
30 \\
\hline
0
\end{array}
$$

Now, find the HCF of 15 litres and 240 litres

$$
\begin{array}{r}
15\overline{)240}(16 \\
15 \\
\hline
90 \\
90 \\
\hline
0
\end{array}
$$

Thus, HCF of 165 litres, 195 litrers and 240 litres = 15 litres

Hence, the largest capactiy of the mug = 15 litres.

14. The floor of a room 6 m 72 cm long and 5 m 46 cm wide has to be paved with square tiles. Find the least number of tiles to pave the floor without leaving any gaps.

Ans. Number of tiles will be least when the size of tiles is greatest.

For the greatest size of tiles we will find the HCF of 672 cm and 546 cm.

$$
\begin{array}{r}
546\overline{)672}(1 \\
546 \\
\hline
126\overline{)546}(4 \\
504 \\
\hline
42\overline{)126}(3 \\
126 \\
\hline
0
\end{array}
$$

$$
\left\{
\begin{array}{l}
\because\ 6\,\text{m}\ 72\,\text{m} \\
\quad = (6\times 100 + 72)\,\text{cm} \\
\quad = 672\,\text{cm} \\
\text{And, } 5\,\text{m}\ 46\,\text{cm} \\
\quad = (5\times 100 + 46)\,\text{cm} \\
\quad = 546\,\text{cm}
\end{array}
\right.
$$

Thus, HCF of 672 cm and 546 cm = 42 cm.

Hence, least number of tiles $= \dfrac{\text{Area of floor}}{\text{Area of one tile}}$

$$\frac{672 \times 546}{42 \times 42} = 16 \times 13 = 208$$

Hence, the least number of tiles to pave the floor without leaving gap = 208.

15. Four ribbons measuring 14 m, 18 m, 22 m and 26 m respectively are to be cut into pieces of equal length.

(*i*) Find the least number of pieces that can be obtained.

(*ii*) What is the length of each piece ?

Ans. HCF of 14, 18, 22 and 26.

HCF of 14 and 18

$$\begin{array}{r} 14\overline{)18}\,(1 \\ 14 \\ \hline 4\overline{)14}\,(3 \\ 12 \\ \hline 2\overline{)4}\,(2 \\ 4 \\ \hline 0 \end{array}$$

Thus, HCF of 14 and 18 = 2

Again, HCF 2 and 22

$$\begin{array}{r} 2\overline{)22}\,(11 \\ 22 \\ \hline 0 \end{array}$$

$\therefore$ HCF of 2 and 22 = 2

Now, HCF of 2and 26,

$$\begin{array}{r} 2\overline{)26}\,(13 \\ 26 \\ \hline 0 \end{array}$$

Hence, HCF of 2 and 26 = 2

(*i*) Least number of pieces that can be obtained

$$= \frac{14}{2} + \frac{18}{2} + \frac{22}{2} + \frac{26}{2} = 7 + 9 + 11 + 13 = 40$$

(*ii*) Thus, length of each piece = 2 m.

16. The product of HCF and LCM of two numbers is 9072. If one of the number is 72, find the other number.

Ans. We know that,

Product of HCF and LCM of two numbers = The product of the numbers. Here, product of HCF and LCM of two numbers is 9072 and one number = 72

Hence, the other number = $\frac{9072}{72} = 126$

17. The HCF of two numbers is 14 and their LCM is 11592, if one of the number is 504, find the other.

Ans. Let the other number be 'y'

Thus, product of two numbers = HCF × LCM

$$y \times 504 = 14 \times 11592$$

$$y = \frac{14 \times 11592}{504} \Rightarrow y = \frac{11592}{36} \Rightarrow y = 322$$

Hence, the other number = 322

18. The product of two numbers is 16184 and their LCM is 952. Find their HCF

Ans. We know that, HCF × LCM = Product of two numbers

Then, $\text{HCF} \times 952 = 16184$

$$\Rightarrow \quad \text{HCF} = \frac{16184}{952}$$

Hence, $\text{HCF} = 17$

19. Find the least number which when divide by 6, 9, 12, 15 and 18 leaves the same remainder 2 in each case.

Ans. The required number = LCM of (6, 9, 12, 15, 18) + 2

i.e.

2	6	9	12	15	18
3	3	9	6	15	9
3	1	3	2	5	3
	1	1	2	5	1

LCM = $2 \times 3 \times 3 \times 2 \times 5 = 180$

Thus, the required number = 180 + 2 = 182

20. Five bells begin to toll together at intervals 6, 7, 8, 9 and 12 seconds. After how much time will they toll together again ?

Ans. LCM of 6, 7, 8, 9 and 12

i.e.

2	6	7	8	9	12
2	3	7	4	9	6
3	1	7	2	9	3
	1	7	2	3	1

Thus, LCM = $2 \times 2 \times 3 \times 7 \times 2 \times 3 = 504$

The required time = 504 seconds = (480 + 24) seconds = 8 minutes 24 seconds.

21. Find the greatest possible length of a rope which can be used to measure exactly the lengths 5 m 13 cm, 7 m 83 cm and 10 m 80 cm.

Ans. The greatest possible length of rope will bethe HCF of 5 m 13 cm, 7 m 83 cm and 10 m 80 cm or 513 cm, 783 cm and 1080 cm. Now HCF of 513, 783 and 1080

3	513
3	171
3	57
19	19
	1

3	783
3	261
3	87
29	29
	1

2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

$513 = 3 \times 3 \times 3 \times 19$

$783 = 3 \times 3 \times 3 \times 29$

$$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

Hence, HCF of 513, 783 and 1080 = $3 \times 3 \times 3 = 27$ cm.

22. Three different containers contain different quantites of mixtures of milk and water whose measurements are 403 kg, 465 kg and 527 kg. What biggest measure must be there to measure all different quantities an exact number of times ?

Ans. The greatest measure will be the HCF of 403 kg, 465 kg and 527 kg

Now HCF of 403 kg, 465 kg and 527 kg

13	403
31	31
	1

3	465
5	155
31	31
	1

17	527
31	31
	1

$$403 = 13 \times 31$$

$$465 = 3 \times 5 \times 31$$

$$527 = 17 \times 31$$

Hence, HCF of 403, 465 and 527 = 31

Thus, greatest measure will be 31 kg.

23. Find the smallest number which when divided by 12, 20, 30 or 60 ; leaves a remainder 5 each time.

Ans.

2	12,	20,	30,	60
2	6,	10,	15,	30
3	3,	5,	15,	15
5	1,	5,	5,	5
	1,	1,	1,	1

Thus, LCM = $2 \times 2 \times 3 \times 5 = 60$

Hence, required smallest number which when divided by 12, 20, 30 or 60; leaves a remainder 5 in each time = 60 + 5 = 65.

24. What is the least number which when increased by 3 is exactly divisible by 27, 35 and 21 ?

Ans.

3	27, 35, 21
7	9, 35, 7
	9, 5, 1

LCM = $3 \times 7 \times 9 \times 5 = 945$

Hence, required least number = $945 - 3 = 942$

25. (*i*) Find the least number that can be divided exactly by all the even numbers between 10 and 20.

(*ii*) Find the least number that can be divided exactly by all odd numbers between 20 and 30.

Ans. (*i*) Even numbers between 10 and 20 are 12, 14, 16, 18

2	12, 14, 16, 18
2	6, 7, 8, 9
3	3, 7, 4, 9
	1, 7, 4, 3

Therefore, LCM of 12, 14, 16 and 18 = $2 \times 2 \times 3 \times 7 \times 4 \times 3$
= 1008

Hence, least number that can be divided exactly by all the even numbers between 10 and 20 = 1008.

(*ii*) Odd numbers between 20 and 30 are 21, 23, 25, 27, 29

3	21, 23, 25, 27, 29
	7, 23, 25, 9, 29

Thus, LCM of 21, 23, 25, 29 and 29

$3 \times 7 \times 23 \times 25 \times 9 \times 29 = 3151575$

Hence, required least number that can be divided exactly by all odd numbers between 20 and 30 = 3151575.

26. What is the least number which when decreased by 5 is divisible by 36, 48, 21 and 28 ?

Ans. LCM of 36, 48, 21, 28

2	36, 48, 21, 28
2	18, 24, 21, 14
3	9, 12, 21, 7
3	3, 4, 7, 7
7	1, 4, 7, 7
2	1, 4, 1, 1
2	1, 2, 1, 1
	1, 1, 1, 1

Thus, LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 1008$

Hence, required number = $1008 + 5 = 1013$.

27. Two persons take steps of 64 cm and 84 cm respectively. If they start in step, how far will they walk before they are in step again ?

Ans. LCM of 64 and 84

2	64, 84
2	32, 42
	16, 21

Thus, LCM of 64 and 84 = $2 \times 2 \times 16 \times 21 = 1344$

Hence, both the two persons will step together again after walking 1344 cm i.e. 13.44 metres.

28. Four bells are ringing at intervals of 12, 16, 24 and 36 minutes. They start ringing simultaneoulsy at 12 O'clock. Find when will they again ring together ?

Ans.

2	12, 16, 24, 36
2	6, 8, 12, 18
2	3, 4, 6, 9
3	3, 2, 3, 9
	1, 2, 1, 3

LCM of 12, 16, 24 and 36 = $2 \times 2 \times 2 \times 3 \times 2 \times 3 = 144$

$\therefore$ Four bells will ring again after 144 minutes *i.e.* after 2 hours 24 minutes

$\because$ The four bells start ringing a 12 O'clock

$\therefore$ These four bells will ring again at = 12 + 2 hours 24 minutes
= 14 hours 24 minutes = 2:24 PM

29. A farmer has 2494 sheep and 2193 lambs. He farms them into flocks; keeping sheep and lambs separate and having the same number of animals in each flock. If thees flocks are as large as possible find :

(*i*) the maximum number of animals in each flock and

(*ii*) total number of flock required for the purpose.

Ans. HCF of 2494 and 2193

$$\begin{array}{r} 2193\overline{)2494}\,(1 \\ 2193 \\ \hline 301\overline{)2193}\,(7 \\ 2107 \\ \hline 86\overline{)301}\,(3 \\ 258 \\ \hline 43\overline{)86}\,(2 \\ 86 \\ \hline 0 \end{array}$$

(*i*) Maximum number of animals in each flock = 43

(*ii*) Total number of flocks required for the purpose

$$= \frac{2193}{43} + \frac{2494}{43} = 51 + 58 = 109.$$

30. Find the largest number which can divide 290, 460 and 552 leaving remainders 4, 5 and 6 respectively.

Ans. Required number = HCF of (290 – 4), (460 – 5) and (552 – 6)
= HCF of 286, 455, 546

Now,

$$
\begin{array}{r}
286\overline{)455}(1 \\
286 \\
\hline
169\overline{)286}(1 \\
169 \\
\hline
117\overline{)169}(1 \\
117 \\
\hline
52\overline{)117}(2 \\
104 \\
\hline
13\overline{)52}(4 \\
52 \\
\hline
0
\end{array}
$$

Now,

$$
\begin{array}{r}
13\overline{)546}(42 \\
52 \\
\hline
26 \\
26 \\
\hline
0
\end{array}
$$

Thus, the required number is 13

31. Find the greatest number that will divide 400, 435 and 541 leaving 9, 10 and 14 as remainders respectively.

Ans. When 400 is divided by that number 9 is left as a remainder so 400 – 9 = 391 is exactly divisible by that number.

Similarly 435 – 10 = 425 and 541 – 14 = 527 are divisible by that number.

Required number is the HCF of the numbers 391, 425 and 527

HCF of 391, 425

$$
\begin{array}{r}
391\overline{)425}(1 \\
391 \\
\hline
34\overline{)391}(11 \\
34 \\
\hline
51 \\
34 \\
\hline
17\overline{)34}(2 \\
34 \\
\hline
0
\end{array}
$$

The HCF of 17 and 527

$$\begin{array}{r} 11\overline{)527}(31 \\ 51 \\ \hline 17 \\ 17 \\ \hline 0 \\ \hline \end{array}$$

Hence, the required number = 17.

32. Two natural numbers are co-prime and their LCM is 6300. If one of the number is 75, find the other number.

Ans. Two natural number are co-prime.

$\therefore$ Their HCF is 1 and LCM is 6300 and one number = 75

Now we know that the Product of HCF and LCM = Product of two numbers

$\therefore$ $6300 \times 1 = 75 \times$ Other number

$\Rightarrow$ Other number $= \dfrac{6300}{75} = 84$

Hence, the number = 84.

33. Can you find two natural numbers with their HCF 12 and LCM 54 ? Justify your answer.

Ans. No, we cannot find two natural numbers with their HCF 12 and LCM 54 because HCF must divide LCM .

34. The LCM of two numbers is 28 times their HCF. The difference of LCM and HCF of these numbers is 810. If one of the number is 120, find the other number.

Ans. Let the HCF of the given number be x.

Given that LCM is 28 times their HCF

$\Rightarrow$ LCM of the number = $28 \times x = 28x$

Further, It is given that LCM – HCF = 810

$\Rightarrow$ $28x - x = 810$

$\Rightarrow$ $27x = 810$ $\Rightarrow$ $x = \dfrac{810}{27} = 30$

Thus, HCF of given number = 30 and their LCM = $28 \times 30 = 840$

The product of HCF and LCM of two numbers equal to the product of the numbers

Here one number = 120 (given)

Thus, the other number = $\frac{30 \times 840}{120} = 30 \times 7 = 210$