7 QUADRILATERAL AND POLYGONS

Q.1.(A) Write in degrees the sum of all interior angles of a :

(i) Hexagon (ii) Septagon (iii) Nonagon (iv) 15-gon

Ans. (i) Sum of interior angles of a hexagon is $(2n-4)$ right angles

$$= (2\times 6-4)\times 90° = (12-4)\times 90° = 8\times 90° = 720°$$

(ii) Sum of interior angles of a septagon is $(2n-4)$ right angles

$$= (2\times 7-4)\times 90° = (14-4)\times 90° = 10\times 90° = 900°$$

(iii) Sum of interior angles of nonagon is $(2n-4)$ right angles

$$= (2\times 9-4)\times 90° = (18-4)\times 90° = 14\times 90° = 260°$$

(iv) Sum of interior angles of a 15-gon is $(2n-4)$ right angles

$$= (2\times 15-4)\times 90° = (30-4)\times 90° = 26\times 90° = 2340°$$

(B) Find the measure, in degrees, of each interior angle of a regular :

(i) Pentagon (ii) Octagon (iii) Decagon (iv) 16-gon

Ans.

(i) Each interior angle of pentagon is $\frac{(2n-4)}{n}$ right angles

$$= \frac{2\times 5-4}{5}\times 90° = \frac{10-4}{5}\times 90° = \frac{6}{5}\times 90° = 108°$$

(ii) Each interior angle of octagon is $\frac{2n-4}{n}$ right angles

$$= \frac{2\times 8-4}{8}\times 90° = \frac{16-4}{8}\times 90° = \frac{12}{8}\times 90° = 135°$$

(iii) Each interior angle of decagon is $\frac{2n-4}{n}$ right angles

$$= \frac{2\times 10-4}{10}\times 90° = \frac{20-4}{10}\times 90° = \frac{16}{10}\times 90° = 144°$$

(iv) Each interior angle of 16-gon is $\frac{2n-4}{n}$ right angles

$$= \frac{2\times 16-4}{16}\times 90° = \frac{32-4}{16}\times 90° = \frac{28}{16}\times 90° = \frac{315}{2} = 157.5°$$

(C) Find the measure, in degrees, of each exterior angle of a regular polygon containing :

(i) 6 sides (ii) 8 sides (iii) 15 sides (iv) 20 sides

Ans. We know that each exterior angle of a polygon of n sides $= \dfrac{360°}{n}$

(i) Each exterior angle of 6 sided polygon $= \dfrac{360°}{6} = 60°$

(ii) Each exterior angle of 8 sides polygon $= \dfrac{360°}{8} = 45°$

(iii) Each exterior angle of 15 sided polygon $= \dfrac{360°}{15} = 24°$

(iv) Each exterior angle of 20 sided polygon $= \dfrac{360°}{20} = 18°$

(D) Find the number of sides of a polygon, the sum of whose interior angles is :

(i) 24 right angles (ii) 1620° (iii) 2880°

Ans. (i) Sum of interior angles of a regular polygons $= 24$ right angles

$\therefore \quad (2n-4) = 24 \Rightarrow 2n = 24 + 4 = 28$

$\therefore \quad n = \dfrac{28}{2} = 14$

Hence, polygon has 14 sides.

(ii) Sum of interior angles of a regular polygon $= 1620°$

$\therefore \quad (2n-4)$ right angles $= 1620° \Rightarrow (2n-4) \times 90° = 1620°$

$\Rightarrow \quad 2n - 4 = \dfrac{1620°}{90°} \Rightarrow 2n - 4 = 18° \Rightarrow 2n = 18 + 4 = 22 \;\therefore\; n = \dfrac{22}{2} = 11$

Hence, polygon has 11 sides.

(iii) Sum of interior angles of a regular polygon $= 2880°$

$\Rightarrow \quad (2n-4)$ right angles $= 2880° \Rightarrow 2n - 4 = \dfrac{2880°}{90°} \Rightarrow 2n - 4 = 32$

$\Rightarrow \quad 2n = 32 + 4 = 36 \;\therefore\; n = \dfrac{36}{2} = 18$

Hence, polygon has 18 sides.

(E) Find the number of sides in a regular polygon, if each of its exterior angles is :

(i) 72° (ii) 24° (iii) (22.5)° (iv) 15°

Ans. We know that each exterior angle of a regular polygon of n sides $= \frac{360°}{n}$

(i) Exterior angle $= 72°$

$\therefore \quad \frac{360°}{n} = 72° \Rightarrow n = \frac{360°}{72°} = 5$. Hence, number of sides of polygon = 5.

(ii) Each exterior angle $= 24°$

$\therefore \quad \frac{360°}{n} = 24 \Rightarrow n = \frac{360°}{24°} = 15$. Hence, number of sides of the regular polygon = 15.

(iii) Each exterior angle $= (22.5)°$

$\therefore \quad \frac{360°}{n} = 22.5° \Rightarrow n = \frac{360°}{22.5°} = \frac{360 \times 10}{225} = 16$.

Hence, number of sides of the regular polygon $= 16$

(iv) Each exterior angle $= 15°$

$\therefore \quad \frac{360°}{n} = 15° \Rightarrow n = \frac{360°}{15°} = 24$.

Hence, number of sides of the regular polygon $= 24$

(F) Find the number of sides in a regular polygon, if each of its interior angles is :

(i) 120° (ii) 150° (iii) 160° (iv) 165°

Ans. We know that each interior angle of a regular polygon of n sides $= \frac{2n-4}{n}$ right angles

(i) Each interior angle $= 120°$

Each interior angle $= \frac{2n-4}{n}$ right angle $= 120°$

$\Rightarrow \quad \frac{2n-4}{n} \times 90° = 120° \Rightarrow \frac{2n-4}{n} = \frac{120°}{90°}$

$\Rightarrow \quad \frac{2n-4}{n} = \frac{4}{3} \Rightarrow 6n - 12 = 4n \Rightarrow 6n - 4n = 12 \Rightarrow 2n = 12 \;\therefore\; n = 6$

Hence, number of sides $= 6$

(ii) Each interior angle $= 150°$

$\therefore \quad \frac{2n-4}{n}$ right angle $= 150° \Rightarrow \frac{2n-4}{n} \times 90° = 150°$

$\Rightarrow \quad \frac{2n-4}{n} = \frac{150°}{90°} = \frac{5}{3} \Rightarrow 6n - 12 = 5n \Rightarrow 6n - 5n = 12 \Rightarrow n = 12$.

Hence, number of sides $= 12$

(iii) Each interior angle $=160°$

$\therefore \quad \frac{2n-4}{n}$ right angles $=160° \Rightarrow \frac{2n-4}{n}\times 90°=160°$

$\Rightarrow \quad \frac{2n-4}{n}=\frac{160°}{90°}=\frac{16}{9} \Rightarrow 18n-36=16n \Rightarrow n=\frac{36}{2}=18$

Hence, number of sides $=18$.

(iv) Each interior angle $=165°$

$\therefore \quad \frac{2n-4}{n}$ right angles $=165° \Rightarrow \frac{2n-4}{n}\times 90°=165°$

$\Rightarrow \quad \frac{2n-4}{n}=\frac{165°}{90°}=\frac{11}{6} \Rightarrow 12n-24=11n \Rightarrow 12n-11n=24 \Rightarrow n=24$

Hence, number of sides $=24$

Q.2.(A) Is it possible to describes a polygon, the sum of whose interior angles is :

(i) 320°　(ii) 540°　(iii) 11 right angles　(iv) 14 right angles

Ans. We know that sum of interior angles of a regular polygon of n sides $=(2n-4)$ right angles.

(i) Sum of interior angles $=320°$

$\therefore \quad (2n-4)$ right angles $=320° \Rightarrow (2n-4)\times 90°=320°$

$\Rightarrow \quad 2n-4=\frac{320°}{90°}=\frac{32}{9} \Rightarrow 2n=\frac{32}{9}+4=\frac{32+36}{9}=\frac{68}{9} \quad \therefore \ n=\frac{68}{9\times 2}=\frac{34}{9}$

Which is in fraction. Hence, it is not possible to describe a polygon.

(ii) Sum of interior angles $=540°$

$\therefore \quad (2n-4)$ right angles $=540° \Rightarrow (2n-4)\times 90°=540°$

$\Rightarrow \quad 2n-4=\frac{540°}{90°}=6 \Rightarrow 2n=6+4=10 \Rightarrow n=\frac{10}{2}=5$

(iii) Sum of interior angles $=11$ right angles

$\therefore \quad (2n-4)$ right angles $=11$ right angles

$\Rightarrow \quad 2n-4=11 \Rightarrow 2n=11+4=15 \Rightarrow n=\frac{15}{2}$

Which is in fraction. Hence, it is not possible to describe a polygons.

(iv) Sum of interior angles $=14$ right angles

$\therefore \quad (2n-4)$ right angles $=14$ right angles

$\Rightarrow \quad 2n-4=14 \Rightarrow 2n=14+4=18 \Rightarrow n=\frac{18}{2}=9$

Hence, it is possible to describe a polygon.

Q.2.(B) Is it possible to have a regular polygon, each of whose exterior angle is :

(i) 32° (ii) 18° (ii) $\frac{1}{8}$ of a right angle (iv) 80°

Ans. We know that exterior angle of a regular polygon of n sides $= \frac{360°}{n}$

(i) Exterior angle $= 32°$

$\therefore \quad \frac{360°}{n} = 32° \Rightarrow n = \frac{360°}{32} = \frac{45}{4}$

Which is in fraction. Hence, it is not possible to have a regular polygon.

(ii) Exterior angle $= 180°$

$\therefore \quad \frac{360°}{n} = 18° \Rightarrow n = \frac{360°}{18°} = 20$

Hence, it is possible to have a regular polygon.

(iii) Exterior angle $= \frac{1}{8}$ of right angle $= \frac{1}{8} \times 90° = \frac{45°}{4}$

$\therefore \quad \frac{360°}{n} = \frac{45°}{4} \Rightarrow n = \frac{360° \times 4}{45} = 32$

Hence, it is possible to have a regular polygon.

(iv) Exterior angle $= 80°$ $\therefore \frac{360°}{n} = 80° \Rightarrow \frac{360°}{80°} = \frac{9}{2}$

Which is in fraction. Hence, it is not possible to have a regular polygon.

Q.2.(C) Is it possible to have a regular polygon, each of whose interior angles is :

(i) 120° (ii) 105° (iii) 175°

Ans. We know that each interior angle of a regular polygon of n sides $= \frac{2n-4}{n}$ right angles.

(i) Interior angle $= 120°$

$\therefore \quad \frac{2n-4}{n}$ right angles $= 120° \Rightarrow \frac{2n-4}{n} \times 90° = 120°$

$\Rightarrow \quad \frac{2n-4}{n} = \frac{120°}{90°} = \frac{4}{3} \Rightarrow 6n - 12 = 4n \Rightarrow 6n - 4n = 12$

$\Rightarrow \quad 2n = 12 \Rightarrow n = 6$. It is possible to have a regular polygon.

(ii) Interior angle $= 105°$

$\therefore \quad \frac{2n-4}{n}$ right angle $= 105° \Rightarrow \frac{2n-4}{n} \times 90° = 105°$

$\Rightarrow \quad \frac{2n-4}{n} = \frac{105°}{90°} \Rightarrow \frac{2n-4}{n} = \frac{7}{6} \Rightarrow 12n - 24 = 7n$

$\Rightarrow \quad 12n - 7n = 24 \Rightarrow 5n = 24 \Rightarrow n = \frac{24}{5}$

Which is fraction. Hence, it is not possible to have a regular polygon.

(iii) Interior angle $= 175°$

$\therefore \quad \frac{2n-4}{n}$ right angle $= 175° \Rightarrow \frac{2n-4}{n} \times 90° = 175°$

$\Rightarrow \quad \frac{2n-4}{n} = \frac{175°}{90°} \Rightarrow \frac{2n-4}{n} = \frac{35}{18} \Rightarrow 36n - 72 = 35n$

$\Rightarrow \quad 36n - 35n = 72 \Rightarrow n = 72$

Hence, it is possible to have a regular polygon.

Q.3. The sum of the interior angles of a polygon is four times the sum of its exterior angles. Find the number of sides in the polygon.

Ans. Let the number of sides of a regular polygon $= n$

Given that :

Sum of interior angles of a regular polygon $= 4\times$ sum of its exterior angles

$\Rightarrow \quad (2n-4) \times 90° = 4 \times 360°$

$\Rightarrow \quad (2n-4) \times 90° = 4 \times 360°$

$\Rightarrow \quad 2n \times 90 - 4 \times 90 = 1440$

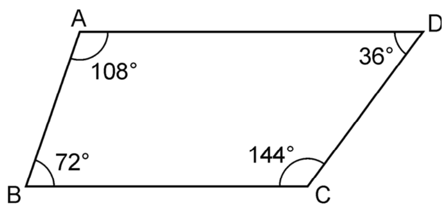
$\Rightarrow \quad 180n - 360 = 1440$

$\Rightarrow \quad 180n = 1440 + 360 \Rightarrow 180n = 1800$

$\Rightarrow \quad n = \frac{1800}{180} \Rightarrow n = 10$. Hence, number of sides of a regular polygon $= 10$

Q.4. The angles of a quadrilateral are in the ratio 3 : 2 : 4 : 1. Find the angles. Assign a special name to the quadrilateral.

Ans. The ratio of angles of quadrilateral $= 3:2:4:1$



Let, the angle of quadrilateral

$= 3x°, 2x°, 4x°, 1x°$

Sum of angles of a quadrilateral $= 360°$

$\Rightarrow \quad 3x° + 2x° + 4x° + x° = 360°$

$\Rightarrow \quad 10x^2 = 360° \Rightarrow 10x = 360 \Rightarrow x = \frac{360°}{10} \Rightarrow x = 36°$

$\therefore$ Angles of quadrilateral are $3x°, 2x°, 4x°, 1x°$

$\Rightarrow$ Angles of quadrilateral are $3\times36°, 2\times36°, 4\times36°, 1\times36°$

$\Rightarrow$ Angles of quadrilateral are $108°, 72°, 144°, 36°$

In the adjoining figure

$\angle A + \angle B = 108° + 72° \Rightarrow \angle A + \angle B = 180°$

i.e. Sum of interior angles on the same side of transversal $AB = 180°$

$\therefore$ AD || BC.

Hence, quadrilateral ABCD is a trapezium.

Q.5. The angles of a pentagon are in the ratio 3 : 4 : 5 : 2 : 4. Find the angles.

Ans. Sum of five angles of a pentagon ABCDE is $(2n-4)$ right angles

$$= (2\times5-4)\times90° = (10-4)\times90° = 6\times90° = 540°$$

The ratio between the angles say $\angle A, \angle B, \angle C, \angle D, \angle E$

$$= 3:4:5:2:4$$

Let $\angle A = 3x$, then $\angle B = 4x$, $\angle C = 5x$, $\angle D = 2x$ and $\angle E = 4x$

$\therefore$ $3x+4x+5x+2x+4x = 540° \Rightarrow 18x = 540° \Rightarrow x = \frac{540°}{18} = 30°$

Hence, $\angle A = 3x = 3\times30° = 90°$, $\angle B = 4x = 4\times30° = 120°$

$\angle C = 5x = 5\times30° = 150°$, $\angle D = 2x = 2\times30° = 60°$, $\angle E = 4x = 4\times30° = 120°$

Q.6. The angles of a pentagon are $(3x+15)°, (x+16)°, (2x+9)°, (3x-8)°$ and $(4x-15)°$ respectively. Find the value of x and hence find the measures of all the angles of the pentagon.

Ans. Let angles of pentagon ABCDE are $(3x+5)°, (x+16)°, (2x+9)°, (3x-8)°$ and $(4x-15)°$.

But the sum of these five angles is $(2n-4)$ right angle

$$= (2\times5-4)\times90° = (10-4)\times90° = 6\times90° = 540°$$

$\therefore$ $3x+5+x+16+2x+9+3x-8+4x-15 = 540°$

$13x+30-23 = 540° \Rightarrow 13x+7 = 540°$

$\Rightarrow$ $13x = 540°-7 = 533° \Rightarrow x = \frac{533}{13} = 41$

$\therefore$ First angle $= 3x+5 = 3\times41+5 = 123+5 = 128°$

Second angle $= x+16 = 41+16 = 57°$

Third angle $= 2x+9 = 2\times41+9 = 82+9 = 91°$

Fourth angle $= 3x-8 = 3\times41-8 = 123-8 = 115°$

Fifth angle $= 4x-15 = 4\times41-15 = 164-15 = 149°$

Q.7. The angles of a hexagon are $2x°, (2x+25)°, 3(x-15)°, (3x-20)°, 2(x+5)°$ and $3(x-15)°$ respectively. Find the value of x and hence find the measures of all the angles of the hexagon.

Ans. Angles a hexagon are $2x°, (2x+25)°, 3(x-15)°, (3x-20)°, 2(x+5)°$ and $3(x-5)°$.

But sum of angles of a hexagon $=(2n-4)$ right angles

$$=(2\times6-4)\times90°=(12-4)\times90°=8\times90°=720°$$

$\therefore \quad 2x+2x+25+3(x-15)+3x-20+2(x+5)+3(x-5)=720°$

$\Rightarrow \quad 2x+2x+25+3x-45+3x-20+2x+10+3x-15=720°$

$\Rightarrow \quad 15x+35-80=720°$

$\Rightarrow \quad 15x=720°+45°\Rightarrow 15x=765°\Rightarrow x=\dfrac{765}{15}=51°$

Hence, first angle $=2x=2\times51°=102°$

Second angle $=2x+25=2\times51°+25°=102°+25°=127°$

Third angle $=3(x-15)=3(51°-15°)=3\times36°=108°$

Fourth angle $=3x-20=3\times51°-20=153°-20°=133°$

Fifth angle $=2(x+5)=2(51+5)=2\times56=112°$

Sixth angle $=3(x-5)=3(51-5)=3\times46=138°$

Hence, angles are 102°, 127°, 108°, 133°, 112° and 138°.

Q.8. Three of the exterior angles of a hexagon are 40°, 52° and 85° respectively and each of the remaining exterior angles is $x°$. Calculate the value of x.

Ans. Sum of exterior angles of a hexagon $=360°$

Three angles are 40°, 52° and 85° and three angles are $x°$ each.

$\therefore \quad 40°+52°+85°+x°+x°+x°=360°\Rightarrow 177°+3x°=360°$

$\Rightarrow \quad 3x°=360°-177°=183° \quad \therefore \quad x=\dfrac{183°}{3}=61°$

Hence, $x=61°$.

Q.9. One angle of an octagon is 100° and other angles are equal. Find the measure of each of the equal angles.

Ans. One angles of an octagon $=100°$

Let each of the other 3 angles $=x°$

But sum of interior angles of an octagon is $(2n-4)$ right angles

$$=(2\times8-4)\times90°=(16-4)\times90°=12\times90°=1080°$$

$\therefore \quad 100+7x=1080\Rightarrow 7x=1080-100 \Rightarrow 7x=980°\Rightarrow x=\dfrac{980}{7}=140°$

Hence, each angle of the remaining angles $=140°$.

Q.10. The interior angle of a regular polygon is double the exterior angle. Find the number of sides in the polygon.

Ans. Let number of sides of a regular polygon $= x$

But sum of interior angle and exterior angle $=180°$

Let each exterior angle $= x°$

Then interior angle $=2x \therefore\ x+2x=180° \Rightarrow\ 3x=180°$

$x=\dfrac{180°}{3}=60°$. Now, $x \times$ exterior angle $=360°$

$x\times 60°=360° \Rightarrow x=\dfrac{360°}{60°}=6$

Hence, number of sides of the regular polygon $=6$.

Q.11. The ratio of each interior angle to each exterior angle of a regular polygon is 7 : 2. Find the number of sides in the polygon.

Ans. Let number of sides of regular polygon $=3$

Ratio of interior angle with exterior angle $=7:2$

Let each interior angle $=7x$ and each exterior angle $=2x$

$\therefore\quad 7x+2x=180°$

$\Rightarrow\quad 9x=180° \Rightarrow x=\dfrac{180°}{9}=20°$

$\therefore\quad$ Each exterior angles $=20x°=2\times 20°=40°$

But sum of exterior angles of a regular polygon of x sides $=360°$

$\Rightarrow\quad x\times 40°=360° \Rightarrow x=\dfrac{360°}{40°}=9$

Hence, number of sides of a regular polygon $=9$.

Q.12. The sum of the interior angles of a polygon is 6 times the sum of its exterior angles. Find the number of sides in the polygon.

Ans. Sum of the exterior angles of a regular polygon of x sides $=360°$

$\therefore\quad$ Sum of its interior angles $=360°\times 6=2160°$

But sum of interior angles of the polygon $=(2x-4)$ right angles

$\therefore\quad (2x-4)\times 90°=2160°$

$\Rightarrow\quad 2x-4=\dfrac{2160°}{90°} \Rightarrow 2x-4=24 \Rightarrow 2x=24+4=28$

$\therefore\quad x=\dfrac{28}{2}=14$. Hence, number of sides $=14$.

Q.13. Two angles of a convex polygon are right angles and each of the other angles is 120°. Find the number of sides of the polygon.

Ans. $\because$ Two angles of a convex polygon $= 90°$ each

$\therefore$ Exterior angles will be $180° - 90° = 90°$ each

Each of other interior angles is 120°.

$\therefore$ Each of exterior angles will be $180° - 120° = 60°$

But the sum of its exterior angles $= 360°$

Let number of sides $= n$

Then $90° + 90° + (n-2) \times 60° = 360° \Rightarrow 180° + (n-2)60° = 360°$

$$60°(n-2) = 360° - 180° = 180° \Rightarrow n - 2 = \frac{180°}{60°} = 3$$

$\therefore$ $n = 3 + 2 = 5$

Hence, number of sides $= 5$.

Q.14. The number of sides of two regular polygons are in the ratio 4 : 5 and their interior angles are in the ratio 15 : 16. Find the number of sides in each polygon.

Ans. Ratio between the sides of two regular polygon $= 4 : 5$

Let number of sides of first polygon $= 4x$

and number of sides the second polygon $= 5x$

$\therefore$ Interior angle of the first polygon $= \dfrac{2 \times 4x - 4}{4x}$ right angles

$\Rightarrow$ $\dfrac{8x-4}{4x} = \dfrac{2x-1}{x}$ right angles and interior angle of second polygon

$= \dfrac{2 \times 5x - 4}{5x}$ right angle

$\Rightarrow$ $\dfrac{10x-4}{5x}$ right angle

$\therefore$ $\dfrac{2x-1}{x} : \dfrac{10x-4}{5x} = 15 : 16 \Rightarrow \dfrac{2x-1}{x} \times \dfrac{5x}{10x-4} = \dfrac{15}{16}$

$\Rightarrow$ $\dfrac{5(2x-1)}{10x-4} = \dfrac{15}{16} \Rightarrow \dfrac{10x-5}{10x-4} = \dfrac{15}{16}$

$\Rightarrow$ $160x - 80 = 150x - 60 \Rightarrow 160x - 150x = -60 + 80 \Rightarrow 10x = 20 \;\therefore\; x = \dfrac{20}{10} = 2$

$\therefore$ Number of sides of the first polygon $= 4x = 4 \times 2 = 8$

and number of sides of the second polygon $= 5 \times 2 = 10$.

Q.15. How many diagonals are there in a

(i) Pentagon (ii) Hexagon (iii) Octagon

Ans. Number of diagonals of a polygon of n sides $= \frac{1}{2}n(n-1)-n$

(i) $\therefore$ Number of diagonals in a pentagon $= \frac{1}{2}\times 5(5-1)-5$ (where $n=5$)

$\Rightarrow \frac{1}{2}\times 5\times 4-5 = 10-5 = 5$

(ii) Number of diagonals in a hexagon $= \frac{1}{2}\times 6(6-1)-6$ (Where $n=6$)

$\Rightarrow \frac{1}{2}\times 6\times 5-6 = 15-6 = 9$

(iii) Number of diagonals in an octagon $= \frac{1}{2}\times 8(8-1)-8$

$\Rightarrow \frac{1}{2}\times 8\times 7-8 = 28-8 = 20$

Q.16. The alternate sides of any pentagon are produced to meet, so as to form a star-shaped figure, shown in the figure. Prove that the sum of measures of the angles at the vertices of the star is 180°.

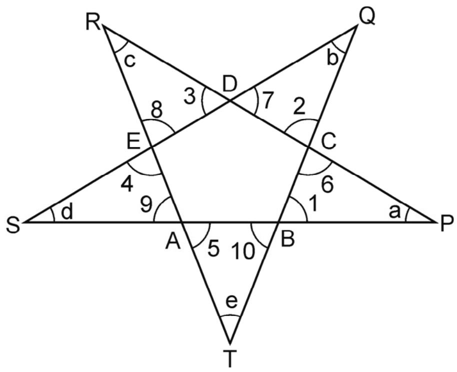
Ans. Given : The alternate sides of a pentagon ABCDE are produced to meet at P, Q, R, S and T so as to form a star shaped figure.

To Prove :

$\angle P + \angle Q + \angle R + \angle S + \angle T = 180°$

or $\angle a + \angle b + \angle c + \angle d + \angle e = 180°$

Proof :



$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$ ...(i)

(Sum of exterior angles of a polygon)

Similarly, $\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 360°$ ...(ii)

But in $\Delta BCP = \angle 1 + \angle b + \angle a = 180°$ ...(iii)

(Sum of angles of a triangle is 180°)

Similarly in ΔCDQ, $\angle 2 + \angle 7 + \angle b = 180°$ ...(iv)

In ΔDER, $\angle 3 + \angle 8 + \angle C = 180°$ ...(v)

In ΔEAS, $\angle 4 + \angle 9 + \angle d = 180°$ ...(vi)

and in ΔABT, $\angle 5 + \angle 10 + \angle e = 180°$ ...(vii)

Adding eqn. (iii), (iv), (v), (vi) and (vii), we get

$\angle 1 + \angle 6 + \angle a + \angle 2 + \angle 7 + \angle b + \angle 3 + \angle 8 + \angle c + \angle 4 + \angle 9 + \angle d + \angle 5 + \angle 10 + \angle e$

$= 180° + 180° + 180° + 180° + 180°$

$\Rightarrow \quad (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5) + (\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10)$

$+ (\angle a + \angle b + \angle c + \angle d + \angle e) = 900°$

$\Rightarrow \quad 360° + 360° + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900°$

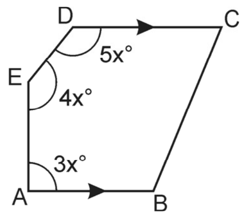
$\Rightarrow \quad 720° + (\angle a + \angle b + \angle c + \angle d + \angle e) = 900°$

$\Rightarrow \quad \angle a + \angle b + \angle c + \angle d + \angle e = 900° - 720° = 180°$

$\Rightarrow \quad \angle P + \angle Q + \angle R + \angle S + \angle T = 180°$

Q.17. In a pentagon ABCDE, AB is parallel to DC and $\angle A : \angle E : \angle D = 3 : 4 : 5$. Find angle E.

Ans. In pentagon ABCDE,



AB || DC and BC is the transversal. [Given]

$\therefore \quad \angle B + \angle C = 180°$ ...(i) [Sum of co-interior angles $= 180°$]

$\angle A : \angle E : \angle D = 3 : 4 : 5$ [Given]

$\therefore$ Let $\angle A = 3x°$, $\angle E = 4x°$ and $\angle D = 5x°$ ...(ii)

Sum of interior angles of n sided polygon $= (2n - 4) \times 90°$ ...(iii)

Sum of interior angles of a pentagon. [Putting $n = 5$]

$= (2 \times 5 - 4) \times 90° = (10 - 4) \times 90°$ ...(iv)

$\Rightarrow \quad 6 \times 90° = 540°$

$\therefore \quad \angle A + \angle B + \angle C + \angle D + \angle E = 540°, \ \angle A + \angle D + \angle E + (\angle B + \angle C) = 540°$

$3x° + 4x° + 5x° + (180°) = 540°$ ...(v)

From eqn. (i), (ii) and (iv), we get

$12x° + 180° = 540°$

$12x° = 540° - 180° \Rightarrow 12x° = 360°$

$\Rightarrow \quad 12x = 360° \Rightarrow x = \dfrac{360°}{12} \Rightarrow x = 30°$

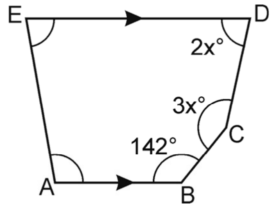
$\angle E = 4x°$ ...(vi) [From eqn. (ii)]

$\angle E = 4 \times 30°$ ...(vii) [From eqn. (v) and (vi)]

$\Rightarrow \quad \angle E = 120°$

Q.18. ABCDE is pentagon in which AB is parallel to ED. If $\angle B = 142°$, $\angle C = 3x°$ and $\angle D = 2x°$, calculate x.

Ans. In pentagon ABCDE



AB || ED (Given)

and AE is the transversal

$\therefore \quad \angle A + \angle E = 180°$ [Sum of co-interior angles $= 180°$] ...(i)

$\angle B = 142°$ (Given) ...(ii)

$\angle C = 3x°$ (Given) ...(iii)

$\angle D = 2x°$ (Given) ...(iv)

Sum of interior angles of n sided polygon $= (2n-4)\times 90°$

Sum of interior angles of pentagon. (Putting $n=5$)

$= (2\times 5-4)\times 90°$

$= (10-4)\times 90° = 6\times 90° = 540°$

$\Rightarrow \angle A+\angle B+\angle C+\angle D+\angle E = 540°$

$(\angle A+\angle E)+\angle B+\angle C+\angle D = 540°$

$180°+142°+3x°+2x° = 540°$ ...(vi)

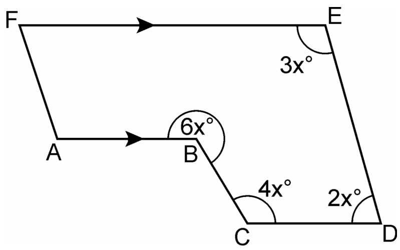
From eqn. (i), (ii), (iii), (iv) and (vi)

$\Rightarrow 322°+5x° = 540° \Rightarrow 5x = 540-322$

$\Rightarrow 5x = 540-322 \Rightarrow 5x = 218$

$\Rightarrow x = \frac{218}{5} \Rightarrow x = 43.6$

Q.19. In a hexagon ABCDEF; side AB is parallel to side EF and $\angle B : \angle C : \angle D : \angle E = 6:4:2:3$. Find angles B and D.



Ans. In hexagon ABCDEF

AB || FE and AF is transversal (Given)

$\therefore \angle A+\angle F = 180°$

[Sum of co-interior angles $= 180°$]

$\angle B:\angle C:\angle D: \angle E = 6:4:2:3$ (Given)

$\therefore$ Let $\angle B = 6x°, \angle C = 4x°, \angle D = 2x°$ and $\angle E = 3x°$

Sum of interior angles of n sided polygon $= (2n-4)\times 90°$

Sum of interior angles of a hexagon $= (2\times 6-4)\times 90° = (12-4)\times 90°$

$= 8\times 90° = 720°$ [Putting $n=6$]

$\Rightarrow \angle A+\angle B+\angle C+\angle D+\angle E+\angle F = 720°$

$\Rightarrow (\angle A+\angle F)+\angle B+\angle C+\angle D+\angle E = 720°$

$\Rightarrow 180°+6x°+4x°+2x°+3x° = 720°$

$\Rightarrow 180°+15x° = 720° \Rightarrow 15x° = 720°-180°$

$\Rightarrow 15x° = 540° \Rightarrow x = \frac{540}{15} \Rightarrow x = 36°$

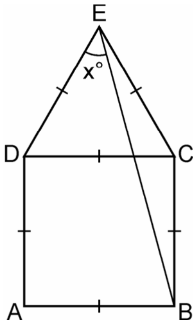
$\angle B = 6x° \Rightarrow \angle B = 6\times 36° \Rightarrow \angle B = 216°$

$\angle D = 2x° \Rightarrow \angle D = 2\times 36° \Rightarrow \angle D = 72°$

Hence, $\angle B = 216°$, and $\angle D = 72°$

Q.20. In the adjoining figure, equilateral ΔEDC surmounts square ABCD. If $\angle DEB = x°$, find the value of x.

Ans. From figure, ABCD is a square and ΔCDE is an equilateral triangle. BE is joined. $\angle DEB = x°$



In ΔBCE, $BC = CE = CD$

$\therefore \quad \angle CBE = \angle CEB$

and $\angle BCE = \angle BCD + \angle DCE = 90° + 60° = 150°$

But $\angle BCE + \angle CBE + \angle CEB = 180°$

(Sum of a triangle is 180°)

$\Rightarrow \quad 150° + \angle CEB + \angle CEB = 180°$

$\Rightarrow \quad 150° + 2\angle CEB = 180°$

$\Rightarrow \quad 2\angle CEB = 180° - 150° = 30° \; \therefore \; \angle CEB = \dfrac{30°}{2} = 15°$

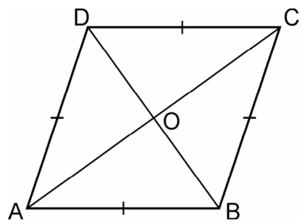
But $\angle CED = 60°$ (Angle of an equilateral triangle is 60°)

$\Rightarrow \quad x° + \angle CEB = 60°$

$\Rightarrow \quad x° + 15° = 60° \Rightarrow x° = 60° - 15° = 45° \; \therefore \; x = 45°$.

Q.21. In the adjoining figure, ABCD is a rhombus whose diagonals intersect at O. if $\angle OAB : \angle OBA = 2:3$, find the angles of ΔOAB.

Ans. ABCD is a rhombus and its diagonal bisect each other at right angles at O.



$\angle OAB : \angle OBA = 2:3$

Let $\angle OAB = 2x$ and $\angle OBA = 3x$

But $\angle AOB = 90°$

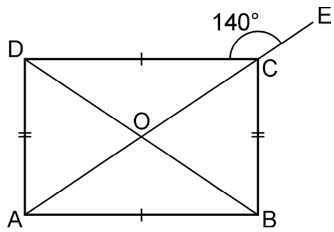
$\therefore \quad \angle OAB + \angle OBA = 90°$

$\Rightarrow \quad 2x + 3x = 90° \Rightarrow 5x = 90°$

$\therefore \quad x = \dfrac{90°}{5} = 18° \; \therefore \; \angle OAB = 2x = 2 \times 18° = 36°$

$\angle OBA = 3x = 3 \times 18° = 54°$ and $\angle AOB = 90°$

Q.22. In the given figure, ABCD is a rectangle whose diagonals intersect at O. Diagonal AC is produced to E and $\angle ECD = 140°$. Find the angles of ΔOAB.



Ans. ABCD is a rectangle and its diagonals AC and BD bisect each other at O.

Diagonal AC is produced to E such that $\angle ECD = 140°$

$\angle ECD + \angle DCO = 180°$ (Linear pair)

$\Rightarrow \quad 140° + \angle DCO = 180°$

$\Rightarrow \quad \angle DCO = 180° - 140° = 40°$

But $OC = OD$ (Half of equal diagonals)

$\therefore \quad \angle CDO = \angle DCO = 40°$

Now $\because$ AB || CD (Opposite sides of a rectangle)

$\therefore \quad \angle OAB = \angle DCO = 40°$ (Alternate angles)

Similarly, $\angle OBA = 40°$

In ΔAOB, $\angle OBA + \angle OAB + \angle AOB = 180°$ (Sum of angles of a triangle is 180°)

$\Rightarrow \quad 40° + 40° + \angle AOB = 180°$

$\Rightarrow \quad 80° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 180° - 80° = 100°$

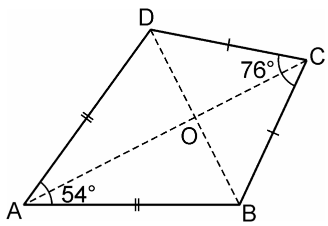
Q.23. In the given figure, ABCD is a kite whose diagonals intersect at O. If $\angle DAB = 54°$ and $\angle BCD = 76°$, calculate : (i) $\angle ODA$, (ii) $\angle OBC$.

Ans. From figure, ABCD is a kite

$\therefore \quad AB = AD, BC = DC$

Its diagonals AC and BD intersect at O.

$\angle DAB = 54°$ and $\angle BCD = 76°$



In ΔBCD,

$\angle CDB = \angle CBD$ ($\because BC = DC$)

But $\angle BCD + \angle CDB + \angle CBD = 180°$

$\Rightarrow \quad 76° + \angle CBD + \angle CDB = 180°$

$\Rightarrow \quad 76° + 2\angle CBD = 180°$

$\Rightarrow \quad 2\angle CBD = 180° - 76° = 104°$

$\therefore \quad \angle CBD = \frac{104°}{2} = 52°$

$\angle OBC = 52°$

In ΔABD, $\angle DAB = 54°$ and $\angle ABD = \angle ADB$

But $\angle DAB + \angle ABD + \angle ADB = 180°$

$\Rightarrow \quad 54° + \angle ADB + \angle ADB = 180°$

$\Rightarrow \quad 54° + 2\angle ADB = 180°$

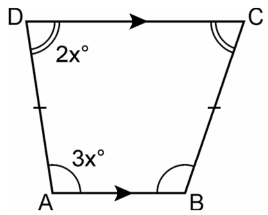
$\Rightarrow \quad 2\angle ADB = 180° - 54° = 126°$

$\therefore \quad \angle ADB = \frac{126°}{2} = 63°$

or $\quad \angle ODA = 63°$

Hence, $\angle ODA = 63°$ and $\angle OBC = 52°$

Q.24. In the given figure, ABCD is an isosceles trapezium in which $\angle CDA = 2x°$ and $\angle BAD = 3x°$. Find all the angles of the trapezium.



Ans. ABCD is an isosceles trapezium in which $AD = BC$ and AB ∥ CD.

$\angle BAD + \angle CDA = 180°$ (Co-interior angles)

$\Rightarrow\ 3x + 2x = 180° \Rightarrow 5x = 180°$

$\therefore\ x = \dfrac{180°}{5} = 36°$

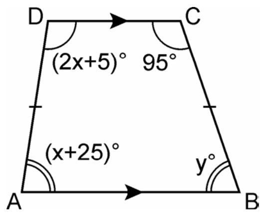
$\therefore\ \angle A = 3x = 3 \times 36° = 108°,\ \angle D = 2x = 2 \times 36° = 72°$

$\because$ ABCD is an isosceles trapezium.

$\therefore\ \angle A = \angle B$ and $\angle C = \angle D$ $\therefore\ \angle B = 108°$ and $\angle C = 72°$

Hence, $\angle A = 108°, \angle B = 108°, \angle C = 72°, \angle D = 72°$.

Q.25. In the given figure, ABCD is a trapezium in which $\angle A = (x + 25)°, \angle B = y°, \angle C = 95°$ and $\angle D = (2x + 5)°$. Find the values of x and y.



Ans. In trapezium ABCD

$\angle A = (x + 25)°, \angle B = y°, \angle C = 95°$ and $\angle D = (2x + 5)°$

$\angle A + \angle D = 180°$ (Co-interior angles)

$\Rightarrow\ (x + 25)° + (2x + 5)° = 180° \Rightarrow x + 25° + 2x + 5° = 180°$

$\Rightarrow\ 3x + 30° = 180° \Rightarrow 3x = 180° - 30° = 150°$

$\therefore\ x = \dfrac{150°}{3} = 50°$

Similarly, $\angle B + \angle C = 180°$

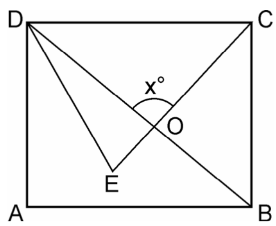
$\Rightarrow\ y + 95° = 180° \Rightarrow y = 180° - 95° = 85°$. Hence, $x = 50°, y = 85°$.

Q.26. DEC is an equilateral triangle in a square ABCD. If BD and CE intersect at O and $\angle COD = x°$, find the value of x.

Ans. ABCD is a square and ΔECD is an equilateral triangle. Diagonal BD and CE intersect each other at O, $\angle COD = x°$.

$\because$ BD is the diagonal of square ABCD

$\therefore\ \angle BDC = \dfrac{90°}{2} = 45° \Rightarrow \angle ODC = 45°$



$\angle ECD = 60°$ (Angle of equilateral triangle) or $\angle OCD = 60°$

Now in ΔOCD,

$\angle OCD + \angle ODC + \angle COD = 180°$

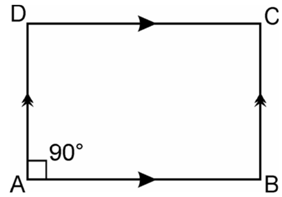
(Sum of angles of a triangle is 180°)

$\Rightarrow\ 45° + 60° + x° = 180°$

$\Rightarrow\ 105° + x° = 180°$

$\therefore \quad x° = 180° - 105° = 75°$

Hence, $x = 75$.



Q.27. If one angle of a parallelogram is 90°, show that each of its angles measures 90°.

Ans. Given : ABCD is a parallelogram and $\angle A = 90°$.

To Prove : Each angle of the parallelogram ABCD is 90°.

Proof : In parallelogram ABCD, $\because \angle A = \angle C$

$\therefore \quad \angle C = 90° \qquad (\because \angle A = 90°)$

But $\angle A + \angle D = 180° \Rightarrow \angle D = 180° - 90° = 90°$ and $\angle B = \angle D$

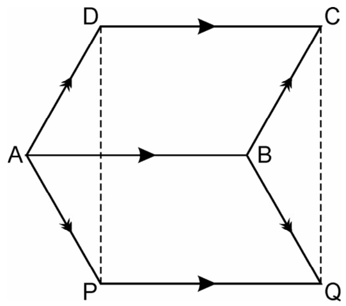
(Opposite angles of a parallelogram)

$\therefore \quad \angle B = 90°$. Hence, $\angle B = \angle C = \angle D = 90°$

Q.28. In the adjoining figure, ABCD and PQBA are two parallelograms. Prove that :

(i) DPQC is a parallelogram. (ii) DP = CQ.

(iii) $\Delta DAP \cong \Delta CBQ$.



Ans. Given : ABCD and PQBA are two parallelogram PD and QC are joined.

To Prove : (i) DPQC is a parallelogram

(ii) DP = CQ (iii) $\Delta DAP \cong \Delta CBO$.

Proof : (i) ABCD and PQBA are parallelogram

DC || AB and AB || PQ(Given)

$\therefore$ DC || PQ

Again DC = AB and AB = PQ (Opposite sides of parallelograms)

$\therefore$ DC = PQ

$\because$ DC = PQ and DC || PQ

$\therefore$ DPQC is a parallelogram.

(ii) $\therefore$ DP = CQ (Opposite sides of parallelogram)

(iii) In ΔDAP and ΔCBQ

DA = CB (Opposite sides of a parallelogram)

AP = BQ (Opposite sides of parallelogram)

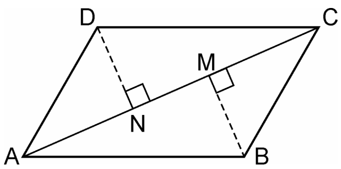
PD = CQ

$\therefore \quad \Delta DAP \cong \Delta CBQ$ (SSS axiom of congruency)

Hence, proved.

Q.29. In the adjoining figure, ABCD is a parallelogram. $BM \perp AC$ and $DN \perp AC$. Prove that :

(i) $\Delta BMC \cong \Delta DNA$. (ii) BM = DN.



Ans. Given : ABCD is a parallelogram.

$BM \perp AC$ and $DN \perp AC$.

To Prove : (i) $\Delta BMC \cong \Delta DNA$ (ii) BM = DN

Proof : In ΔBMC and ΔDNA

BC = AD (Opposite sides of a parallelogram)

$\angle M = \angle N = 90°$

$\angle BCM = \angle DAN$ (Alternate angles)

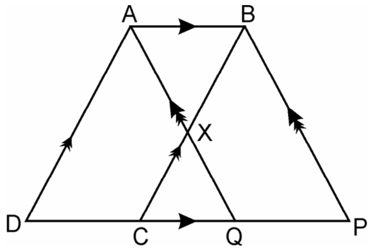
(i) $\therefore$ $\Delta BMC \cong \Delta DNA$ (AAS axiom of congruency)

(ii) $\therefore$ BM = DN (CPCT)

Q.30. In the adjoining figure, ABCD is a parallelogram and X is the mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram AQPB is completed. Prove that :

(i) $\Delta ABX \cong \Delta QCX$.

(ii) DC = CQ = QP.



Ans. Given : ABCD is a parallelogram X is mid-point of BC.

AX is joined and produced to meet DC produced at Q. From B, BP is drawn parallel to AQ so that AQPB is a parallelogram.

To Prove : (i) $\Delta ABX \cong \Delta QCX$.

(ii) DC = CQ = QP.

Proof : (i) In ΔABX and ΔQCX.

XB = XC ($\because$ X is mid-point of BC)

$\angle AXB = \angle CXQ$ (Vertically opposite angles)

$\angle BAX = \angle XQC$ (Alternate angles)

$\therefore$ $\Delta ABX \cong \Delta QCX$ (ASA axiom of congruency)

(ii) In parallelogram ABCD,

AB = DC ...(i) (Opposite sides of a parallelogram)

Similarly, in parallelogram AQPB

AB = QP ...(ii)

$\therefore$ From eqn. (i) and (ii), we get

DC = QP ...(iii)

In ΔBCP,

X is mid-point of BC and AQ $\parallel$ BP $\therefore$ Q is mid-point of CP.

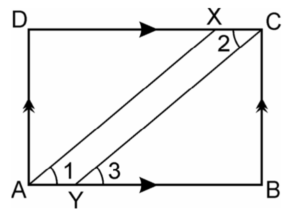
$\Rightarrow$ CQ = QP ...(iv)

From eqn. (iii) and (iv), we get
DC = QP = CQ or DC = CQ = QP.

Q.31. In the adjoining figure, ABCD is a parallelogram. Line segments AX and CY bisect $\angle A$ and $\angle C$ respectively. Prove that :

(i) $\Delta ADX \cong \Delta CBY$ (ii) AX = CY

(iii) AX || CY (iv) AYCX is a parallelogram



Ans. Given : ABCD is a parallelogram. Line segments AX and CY bisect $\angle A$ and $\angle C$ respectively.

To Prove : (i) $\Delta ADX \cong \Delta CBY$ (ii) AX = CY
(iii) AX || CY (iv) AYCX is a parallelogram.

Proof : (i) In ΔADX and ΔCBY.

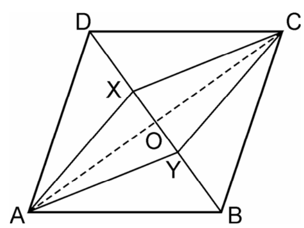
AD = BC (Opposite sides of a parallelogram)
$\angle D = \angle B$ (Opposite angles of the parallelogram)
$\angle DAX = \angle BCY$ (Half of equal angles A and C)
$\therefore$ $\Delta ADX \cong \Delta CBY$ (ASA axiom of congruency)
(ii) $\therefore$ AX = CY (CPCT)
(iii) $\angle 1 = \angle 2$ (Half of equal angles)
But $\angle 2 = \angle 3$ (Alternate angles)
$\therefore$ $\angle 1 = \angle 3$
But these are corresponding angles.
$\therefore$ AX || CY
(iv) $\because$ AX = CY and AX || CY
$\therefore$ AYCX is a parallelogram.

Q.32. In the given figure, ABCD is a parallelogram and X, Y are points on diagonal BD such that DX = BY. Prove that CXAY is a parallelogram.

Ans. Given : ABCD is a parallelogram. X and Y are points on diagonal BD such that DX = BY.

To Prove : CXAY is a parallelogram.

Construction : Join AC meeting BD at O.



Proof : $\because$ AC and BD are the diagonals of the parallelogram ABCD.
$\therefore$ AC and BD bisect each other at O.
$\therefore$ AO = OC and BO = OD
But DX = BY (Given)
$\therefore$ DO – DX = OB – BY
$\Rightarrow$ OX = OY
Now in quadrilateral CXAY, diagonals AC and XY bisect each other at O.

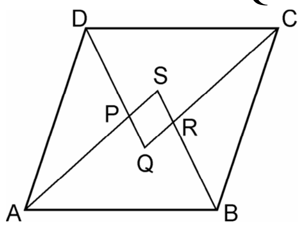
$\therefore$ CSAY is a parallelogram.

Q.33. Show that the bisectors of the angles of a parallelogram enclose a rectangle.

Ans. Given : ABCD is a parallelogram.

Bisectors of $\angle A$ and $\angle B$ meet at S and bisectors of $\angle C$ and $\angle D$ meet at Q.

To Prove : PQRS is a rectangle.



Proof : $\because\ \angle A + \angle B = 180° \ \therefore\ \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

$\Rightarrow\ \angle SAB = \angle SBA = 90°$

$\therefore$ In $\Delta ASB,\ \angle ASB = 90°$

Similarly we can prove that $\angle CQD = 90°$

Again $\angle A + \angle D = 180° \ \therefore\ \frac{1}{2}\angle A + \frac{1}{2}\angle D = 90°$

$\Rightarrow\ \angle PAD = \angle PDA = 90° \ \therefore\ \angle APD = 90°$

But $\angle SPQ = \angle APD$ (Vertically opposite angles)

$\therefore\ \angle SPQ = 90°$

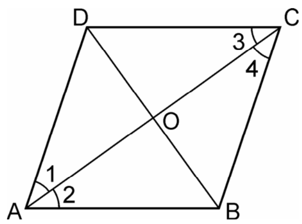
$\because$ Similarly, we can prove that $\angle SRQ = 90°$

$\because$ In quadrilateral PQRS, its each angle is of 90°.

Hence, PQRS is a rectangle.

Q.34. If a diagonal of a parallelogram bisects one of the angles of the parallelogram, prove that it also bisects the second angle and then the two diagonals are perpendicular to each other.

Ans. Given : In parallelogram ABCD, diagonal AC bisects $\angle A$. BD is joined meeting AC at O.



To Prove : (i) AC bisects $\angle C$.

(ii) Diagonal AC and BD are perpendicular to each other.

Proof : In parallelogram ABCD $\because$ AB || DC

$\therefore\ \angle 1 = \angle 4$

and $\angle 2 = \angle 3$ (Alternate angles)

But $\angle 1 = \angle 2$ (Given)

$\therefore\ \angle 3 = \angle 4$

Hence, AC bisects $\angle C$ also. Similarly we can prove that diagonal BD will also bisect the $\angle B$ and $\angle D$. $\therefore$ ABCD is a rhombus.

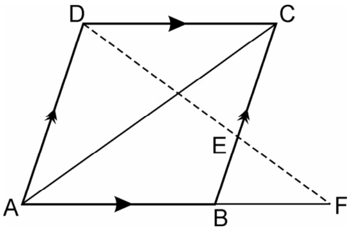
But diagonals of a rhombus bisect each other at right angles.

$\therefore$ AC and BD are perpendicular to each other.

Q.35. In the given figure, ABCD is a parallelogram and E is the mid-point of BC. If DE and AB produced meet at F, prove that AF = 2AB.

Ans. Given : ABCD is a parallelogram. E is mid-point of BC. DE and AB are produced to meet at F.

To Prove : AF = 2AB.



Proof : In parallelogram in ΔDEC and ΔFEB

$CE = EB$ ($\because$ E is mid-point of BC)

$\angle DEC = \angle BEF$ (Vertically opposite angles)

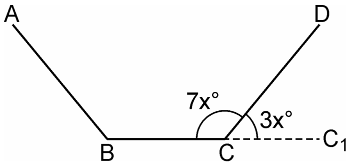
$\angle DCE = \angle EBF$ (Alternate angles)

$\therefore$ $\Delta DEC \cong \Delta FEB$ (AAS axiom of congruency)

$\therefore$ $CD = BF$ (CPCT)

But $AB = CD$ (Opposite sides of a parallelogram)

Q.36. If the ratio of interior angle to the exterior angle of a regular polygon is 7 : 2. Find the number of sides in the polygon.



Ans. Ratio of interior angle to the exterior angle of regular polygon $= 7:2$

Let the interior angle BCD $= 7x°$

Let the exterior angle $DCC_1 = 2x°$

BCC_1 is a straight line

$\therefore$ $\angle BCD + \angle DCC_1 = 180° \Rightarrow 7x° + 2x° = 180°$

$$9x° = 180° \Rightarrow x = \frac{180°}{9} \Rightarrow x = 20°$$

$\therefore$ Interior angle $= 7x° = 7 \times 20° = 140°$, Exterior angle $= 2x°$

$\Rightarrow$ Exterior angle $= 2 \times 20°$, $\Rightarrow$ Exterior angle $= 40°$

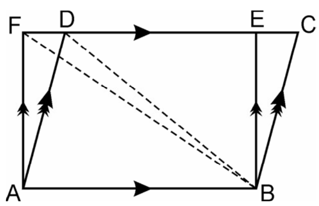
Hence, number of sides.

$$\Rightarrow \frac{360°}{n} = 40° \quad \Rightarrow \frac{360°}{n} = 40° \quad \Rightarrow 360 = 40n \quad \Rightarrow 40n = 360$$

$$\Rightarrow n = \frac{360}{40} \quad \Rightarrow n = 9$$

Hence, number of sides of regular polygon is 9.

Q.37. In the given figure, the area of parallelogram ABCD is 90 cm^2. State giving reasons : (i) ar. (||gm ABEF) (ii) ar. (ΔABD) (iii) ar. (ΔBEF).



Ans. Area of ||gm $ABCD = 90\ cm^2$

AF || BE are drawn and BD and BF are joined.

$\therefore$ ABEF is a parallelogram.

(i) Now ||gm ABCD and ||gm ABEF are on the same base and between the same parallel lines.

$\therefore$ Area of ||gm ABCD = area of || gm ABEF

But area of || gm ABCD $= 90\text{ cm}^2$ $\therefore$ Area of || gm ABEF $= 90\text{ cm}^2$

(ii) $\because$ BD and BF are the diagonals of || gm ABCD and || gm ABEF respectively and diagonals of a || gm bisect it into two triangles of equal area.

$\therefore$ Area $(\Delta ABD) = \frac{1}{2}$area (||gm ABCD)

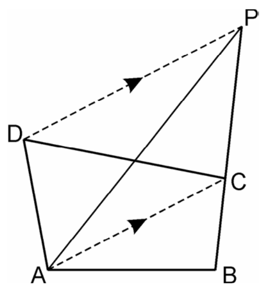
$$= \frac{1}{2} \times 90\text{ cm}^2 = 45\text{ cm}^2$$

(iii) and area $(\Delta BEF) = \frac{1}{2}$ area (||gm ABEF)

$$= \frac{1}{2} \times 90\text{ cm}^2 = 45\text{ cm}^2$$

Q.38. In the given figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P. Prove that : ar. (ΔABP) = ar. (quad. ABCD).

Ans. Given : In quad. ABCD, a line through D is drawn parallel to AC and meets BC produced in P.



To Prove : Area (ΔABP) = Area (quadrilateral ABCD)

Proof : In quadrilateral ABCD,

$\because$ AC || PD and ΔACD and ΔACP are on the same base AC and between the same parallel lines.

$\therefore$ Area (ΔACD) = Area (ΔACP)

Adding area (ΔABC) both sides,

Area (ΔACD) + Area (ΔABC) = Area (ΔACP) + Area (ΔABC)

$\Rightarrow$ Area (quad. ABCD) = Area (ΔABP)

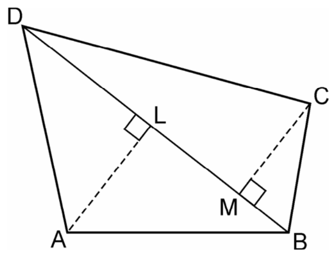
or ar. (ΔABP) = ar. (quadrilateral ABCD)

Q.39. ABCD is a quadrilateral. If $AL \perp BD$ and $CM \perp BD$, prove that: ar. (quad. ABCD) = $\frac{1}{2} \times BD \times (AL + CM)$.

Ans. Given : In quadrilateral ABCD, $AL \perp BD$ and $CM \perp BD$.

To Prove : ar. (quad. ABCD) $= \frac{1}{2} \times BD \times (AL + CM)$

Proof : In quadrilateral ABCD,



$$\text{ar. }(\Delta ABD)=\frac{1}{2}\text{base}\times\text{altitude}=\frac{1}{2}BD\times AL \quad ...(i)$$

Again, $\text{ar. }(\Delta BCD)=\frac{1}{2}\times BD\times CM \quad ...(ii)$

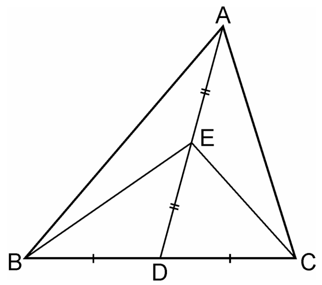
Adding (i) and (ii), we get

$$\text{ar. }(\Delta ABD)+\text{ar. }(\Delta BCD)=\frac{1}{2}BD\times AL+\frac{1}{2}BD\times CM$$

$$\Rightarrow \text{ar. (quad. }ABCD)=\frac{1}{2}BD(AL+CM)$$

Q.40. In the given figure, D is the mid-point of BC and E is the mid-point of AD. Prove that : $\text{ar.}(\Delta ABE)=\frac{1}{4}\text{ar.}(\Delta ABC)$.

Ans. Given : In ΔABC, D is mid-point of BC and E is mid-point on AD. CE and BE are joined.,



To Prove : $\text{ar. }(\Delta ABE)=\frac{1}{4}\text{ar. }(\Delta ABC)$.

Proof : In ΔABC, AD is the median

$\therefore \quad \text{ar. }(\Delta ABD)=\text{ar. }(\Delta ACD)$

$\quad =\frac{1}{2}\text{ar. }(\Delta ABC) \quad ...(i)$

Again in ΔABD, BE is the median

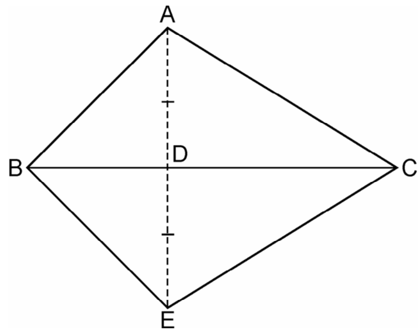
$\therefore \quad \text{ar. }(\Delta ABE)=\text{ar. }(\Delta EBD)=\frac{1}{2}\text{ar }(\Delta ABD)$

$\quad =\frac{1}{2}\times\frac{1}{2}\text{ar. }(\Delta ABC) \quad$ [From (i)]

$\quad =\frac{1}{4}\text{ar. }(\Delta ABC)$.

Q.41. In the given figure, a point D is taken on side BC of ΔABC and AD is produced to E, making DE = AD. Show that : $\text{ar.}(\Delta BEC)=\text{ar.}(\Delta ABC)$.

Ans. Given : In ΔABC, D is any point on BC, AD is joined and produced to E such that DE = AD. BE and CE are joined.



To Prove : $\text{ar. }(\Delta BEC)=\text{ar. }(\Delta ABC)$.

Proof : In ΔABC, $\because$ AD = DE

$\therefore$ D is mid-point of AE.

In ΔABE, BD is the median

$\therefore$ ar. $(\Delta BDE) =$ ar. (ΔABD) ...(i)

Similarly, in ΔACE, CD is the median

$\therefore$ ar. $(\Delta CDE) =$ ar. (ΔACD) ...(ii)

Adding eqn. (i) and (ii), we get

ar. $(\Delta BDE) +$ ar. $(\Delta CDE) =$ ar. $(\Delta ABD) +$ ar. (ΔACD)

$\Rightarrow$ ar. $(\Delta BEC) =$ ar. (ΔABC).

Q.42. If the medians of a ΔABC intersect at G, show that :

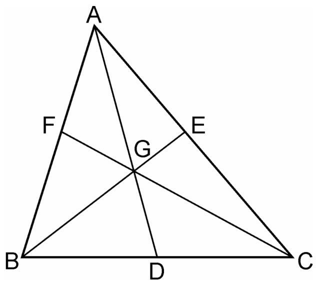
ar. $(\Delta AGB) =$ ar. $(\Delta AGC) =$ ar. $(\Delta BGC) = \frac{1}{3}$ ar. (ΔABC)

Ans. Given : In ΔABC, AD, BE and CF are the medians of the sides BC, CA and AB respectively intersecting at the point G.

To Prove : ar. $(\Delta AGB) =$ ar. $(\Delta AGC) =$ ar. $(\Delta BGC) = \frac{1}{3}$ ar. (ΔABC)

Proof : In ΔABC, AD is the median

$\therefore$ ar. $(\Delta ABD) =$ ar. (ΔACD) ...(i)



Again in ΔGBC, GD is the median

$\therefore$ ar. $(\Delta GBD) =$ ar. (ΔGCD) ...(ii)

Subtracting (ii) from (i), we get

ar. $(\Delta ABD) -$ ar. $(\Delta GBD) =$ ar. $(\Delta ACD) -$ ar. (ΔGCD)

$\Rightarrow$ ar. $(\Delta AGB) =$ ar. (ΔAGC) ...(iii)

Similarly, we can prove that

ar. $(\Delta AGC) =$ ar. (ΔBGC) ...(iv)

From eqn. (iii) and (iv), we get

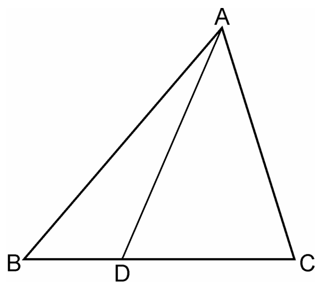
ar. $(\Delta AGB) =$ ar. $(\Delta AGC) =$ ar. (ΔBGC)

But ar. $(\Delta AGB) +$ ar. $(\Delta AGC) +$ ar. $(\Delta BGC) =$ ar. (ΔABC)

ar. $(\Delta AGB) =$ ar. $(\Delta AGC) =$ ar. $(\Delta BGC) = \frac{1}{3}$ ar. (ΔABC)

Q.43. D is a point on base BC of a ΔABC such that 2BD = DC. Prove that :

ar. $(\Delta ABD) = \frac{1}{3}$ ar. (ΔABC).



Ans. Given : In ΔABC, D is a point on BC such that 2 BD = DC.

To Prove : ar. $(\Delta ABD) = \frac{1}{3}$ ar. (ΔABC).

Proof : In ΔABC, $\because$ $2BD = DC$ $\Rightarrow$ $\frac{BD}{DC} = \frac{1}{2}$

$\Rightarrow$ $BD : DC = 1 : 2$

$\therefore$ ar. (ΔABD) : ar. $(\Delta ADC) = 1 : 2$

But ar. (ΔABD) + ar. (ΔADC) = ar. (ΔABC)

$\Rightarrow$ ar. $(\Delta ABD) + 2$ ar. (ΔABD) = ar. (ΔABC)

$\Rightarrow$ 3 ar. (ΔABD) = ar. (ΔABC)

$\Rightarrow$ ar. $(\Delta ABD) = \frac{1}{3}$ ar. (ΔABC).

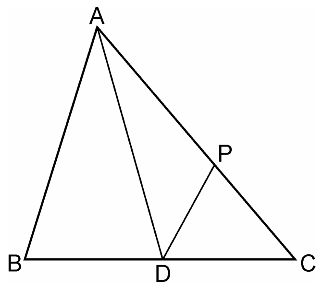
Q.44. In the given figure, AD is a median of ΔABC and P is a point on AC such that : ar. (ΔADP) : ar. $(\Delta ABD) = 2 : 3$. Find :

(i) AP : PC **(ii) ar. (ΔPDC) : ar. (ΔABC).**

Ans. Given : In ΔABC, AD is median of the triangle, P is a point on AC such that :

ar. (ΔADP) : ar. $(\Delta ABD) = 2 : 3$

Now we have



To Prove : (i) AP : PC

(ii) ar. (ΔPDC) : ar. (ΔABC).

Proof : (i) In ΔABC, AD is the median.

$\therefore$ ar. (ΔABD) = ar. (ΔADC) ...(i)

$\because$ ar. (ΔADP) : ar. $(\Delta ABD) = 2 : 3$

$\Rightarrow$ ar. (ΔADP) : ar. $(\Delta ADC) = 2 : 3$

$\Rightarrow$ ar. (ΔADC) : ar. $(\Delta ADP) = 3 : 2$

$\Rightarrow$ $\frac{\text{ar. }(\Delta ADC)}{\text{ar. }(\Delta ADP)} = \frac{3}{2}$

$\Rightarrow$ $\frac{\text{ar. }(ADC)}{\text{ar. }(ADP)} - 1 = \frac{3}{2} - 1$ (Subtracting both sides)

$\Rightarrow$ $\frac{\text{ar. }(\Delta ADC) - \text{ar. }(\Delta ADP)}{\text{ar. }(\Delta ADP)} = \frac{1}{2}$

$\Rightarrow$ $\frac{\text{ar. }(\Delta ADP)}{\text{ar. }(\Delta PDC)} = \frac{2}{1}$ ...(ii)

$\Rightarrow$ ar. (ΔADP) : ar. $(\Delta PDC) = 2 : 1$ $\therefore$ $AP : PC = 2 : 1$

(ii) Now $\frac{\text{ar. }(\Delta ADP)}{\text{ar. }(\Delta PDC)} = \frac{2}{1}$ [From (ii)]

Adding 1 both sides, we get

$$\frac{\text{ar. }(\Delta ADP)}{\text{ar. }(\Delta PDC)}+1=\frac{2}{1}+1$$

$$\frac{\text{ar. }(\Delta ADP)+\text{ar. }(\Delta PDC)}{\text{ar. }(\Delta PDC)}=\frac{2}{1}+1$$

$$\frac{\text{ar. }(\Delta ADC)}{\text{ar. }(\Delta PDC)}=\frac{3}{1}$$

But $\text{ar. }(\Delta ADC)=\text{ar. }(\Delta ABD)$ [From (i)]

$$\therefore \quad \frac{\text{ar. }(\Delta ADB)}{\text{ar. }(\Delta PDC)}=\frac{3}{1}\Rightarrow\frac{\text{ar. }(\Delta PDC)}{\text{ar. }(\Delta ABD)}=\frac{1}{3}$$

But $\text{ar. }(\Delta ABD)=\frac{1}{2}\text{ar. }(\Delta ABC)$

$$\therefore \quad \frac{\text{ar. }(\Delta PDC)}{\frac{1}{2}\text{ar. }(\Delta ABC)}=\frac{1}{3}$$

$$\Rightarrow \quad \frac{2\text{ar. }(\Delta PDC)}{\text{ar. }(\Delta ABC)}=\frac{1}{3}$$

$$\Rightarrow \quad \frac{\text{ar. }(\Delta PDC)}{\text{ar. }(\Delta ABC)}=\frac{1}{3\times 2}=\frac{1}{6}$$

Hence, $\text{ar. }(\Delta PDC):\text{ar. }(\Delta ABC)=1:6$

Q.45. In the given figure, P is a point on side BC of ΔABC such that BP : PC = 1 : 2 and Q is a point on AP such that PQ : QA = 2 : 3. Show that : $\textbf{ar.}(\Delta AQC):\textbf{ar.}(\Delta ABC) = 2:5$.

Ans. Given : In ΔABC, P is a point on BC such that BP : PC = 1 : 2. Q is a point on AP such that PQ : QA = 2 : 3.

To Prove : $\text{ar. }(\Delta AQC):\text{ar. }(\Delta ABC)=2:5$

Proof : In ΔABC, P is a point on BC such that

BP : PC = 1 : 2

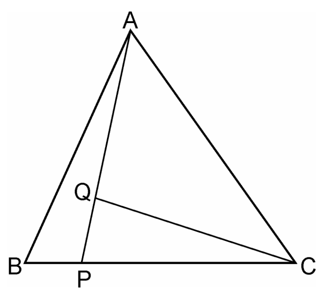
$\therefore$ $\text{ar. }(\Delta APB):\text{ar. }(\Delta APC)=1:2$, $\text{ar. }(\Delta APC)=\frac{2}{3}\text{ar. }(\Delta ABC)$

In ΔAPC,

Q is a point on AP such that PQ : QA = 2 : 3

$\Rightarrow$ $\text{ar. }(\Delta AQC):\text{ar. }(\Delta PQC)=3:2$

or $\text{ar. }(\Delta AQC)=\frac{3}{5}\text{ar. }(\Delta APC)$



$= \frac{3}{5} \times \frac{2}{3} \times \text{ar. } (\Delta ABC)$

$= \frac{2}{5} \text{ar. } (\Delta PBC)$

$\Rightarrow \quad \frac{\text{ar. } (\Delta AQC)}{\text{ar. } (\Delta ABC)} = \frac{2}{5}$

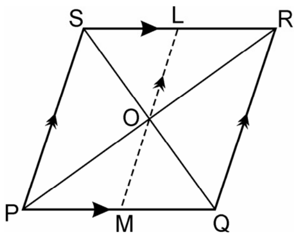
$\therefore \quad \text{ar. } (\Delta AQC) : \text{ar. } (\Delta ABC) = 2 : 5$

Q.46. In the given figure, diagonals PR and QS of the parallelogram PQRS intersect at point O and LM is parallel to PS. Show that :

(i) 2 ar. (ΔPOS) = ar. (|| gm PMLS)

(ii) ar. (ΔPOS) + ar. (ΔQOR) = $\frac{1}{2}$ ar. (|| gm PQRS)

(iii) ar. (ΔPOS) + ar. (ΔQOR) = ar. (ΔPOQ) + ar. (ΔSOR)



Ans. Given : PQRS is a || gm in which diagonals PR and QS intersect at O. LM || PS.

To Prove : (i) 2 ar. (ΔPOS) = ar. (||gm PMLS)

(ii) $\text{ar. } (\Delta POS) + \text{ar. } (\Delta QOR) = \frac{1}{2} \text{ar. } (\|\text{gm } PQRS)$

(iii) ar. (ΔPOS) + ar. (ΔQOR) = ar. (ΔPOQ) + ar. (ΔSOR)

Proof : In parallelogram PQRS

(i) PS || LM (Given)

and PM || SL [∵ PQ || SR; opposite sides of || gm are parallel]

∴ PMLS is a || gm

ΔPOS and || gm PMLS are on the same base PS and between the same parallel lines PS and LM.

$\therefore \quad \text{ar. } (\Delta POS) = \frac{1}{2} \text{ar. } (\|\text{gm } PMLS)$

⇒ 2ar. (ΔPOS) = ar. (|| gm PMLS) ...(i)

(ii) QR || LM and MQ || LR [∵ LM || PS and PS || QR] [∵ PQ || SR]

∴ MQRL is a || gm.

∴ QOR and || gm MQRL are on the same base QR and between the same || lines QR and LM.

∴ 2ar. (ΔQOR) = ar. (||gm MQRL) ...(ii)

Adding (i), (ii), we get

2ar.(ΔPOS) + 2ar. (ΔQOR) = ar. (||gm PMLS) + ar. (||gm MQRL)

⇒ 2[ar. (ΔPOS) + ar. (ΔQOR) = ar. (||gm PQRS)

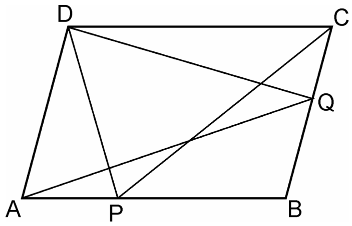
$\Rightarrow$ ar (ΔPOS) + ar. $(\Delta QOR) = \frac{1}{2}$ ar. (||gm PQRS) ...(iii)

(iii) As in part (ii), we can prove that

ar. (ΔPOQ) + ar. $(\Delta SOR) = \frac{1}{2}$ ar. (||gm PQRS) ...(iv)

From (iii) and (iv), we get

ar. (ΔPOS) + ar. (ΔQOR) = ar. (ΔPOQ) + ar. (ΔSOR)



Q.47. In parallelogram ABCD. P is a point on side AB and Q is a point on side BC. Prove that :

(i) ΔCPD and ΔAQD are equal in area.

(ii) ar. (ΔAQD) = ar. (ΔAPD) + ar. (ΔCPB)

Ans. Given : || gm ABCD in which P is a point on AB and Q is a point on BC.

To Prove : (i) ar. (ΔCPD) = ar. (ΔAQD)

(ii) ar. (ΔAQD) = ar. (ΔAPD) + ar.(ΔCPB)

Proof : In parallelogram ABCD,

ΔCPD and || gm ABCD are the same base CD and between the same parallels AB and CD.

$\therefore$ ar. $(\Delta CPD) = \frac{1}{2}$ ar. (||gm ABCD) ...(i)

ΔAQD and || gm ABCD are on the same base AD and between the same || lines AD and BC.

$\therefore$ ar. $(\Delta AQD) = \frac{1}{2}$ ar. (|| gm ABCD) ...(ii)

From (i) and (ii), we get

ar. (ΔCPD) = ar. (ΔAQD)

(ii) ar. $(\Delta AQD) = \frac{1}{2}$ ar. (|| gm ABCD)

$\Rightarrow$ 2ar. (ΔAQD) = ar. (|| gm ABCD)

ar. (ΔAQD) + ar. (ΔAQD) = ar. (|| gm ABCD) ...(iii)

But, ar. (ΔAQD) = ar. (ΔCPD) ...(iv)

From (iii) and (iv), we get

ar. (ΔAQD) + ar. (ΔCPD) = ar. (||gm ABCD)

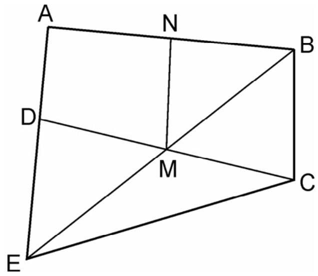
$\Rightarrow$ ar. (ΔAQD) + ar. (ΔCPD) = ar. (ΔAPD) + ar. (ΔCPD) + ar. (ΔCPB)

$\Rightarrow$ ar. (ΔAQD) = ar. (ΔAPD) + ar. (ΔCPB)

Q.48. In the given figure, M and N are the mid-points of the sides DC and AB respectively of the parallelogram ABCD. If the area of parallelogram ABCD is 48 cm^2;

(i) state the area of the triangle BEC.

(ii) name the parallelogram which is equal in area to the triangle BEC.



Ans. Given : ABCD is || gm in which M and N are the mid-points of sides DC and AB respectively. BM is joined and produced to meet AD produced at E. CE is joined : Ar. (|| gm ABCD) = 48 cm^2.

To Prove : (i) To find ar. (ΔBEC)

(ii) To name the || gm which is equal in area to the ΔBEC.

Proof : In parallelogram ABCD,

(i) ΔBEC and || gm ABCD are on the same base BC and between the same || lines AD and BC.

$\therefore$ ar. (ΔBEC) $= \frac{1}{2}$ ar. (|| gm ABCD) ...(i)

But, ar. (|| gm ABCD) = 48 cm^2 (Given) ...(ii)

From eqn. (i) and (ii), we get

ar. (ΔBEC) $= \frac{1}{2} \times 48 \text{ cm}^2 = 24 \text{ cm}^2$

(ii) M and N are mid-points of AB and CD.

In ΔABE, MN will be || to AE. Also, MN bisects the || gm ABCD in two equal parts. Now, MN || BC and BN || MC. Therefore, BNMC is a || gm.

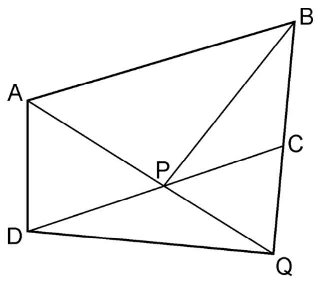
$\therefore$ ar. (|| gm BNMC) $= \frac{1}{2}$ ar. (|| gm ABCD) ...(iii)

From (i) and (iii), we get

ar. (ΔBEC) = ar. (|| gm BNMC)

$\therefore$ BNMC is the required || gm which is equal in area to ΔBEC.

Q.49. ABCD is a parallelogram, a line through A cuts DC at point P and BC produced at Q. Prove that triangle BCP is equal in area to triangle DPQ.



Ans. Given : || gm ABCD in which a line through A cuts DC at P and BC produced at Q.

To Prove : ar. (ΔBCP) = ar. (ΔDPQ)

Proof : ΔAPB and || gm ABCD are on the same base

AB and between the same || lines AB and CD.

$\therefore$ ar. $(\Delta APB) = \frac{1}{2}$ ar. (|| gm ABCD) ...(i)

ΔADQ and || gm ABCD are on the same base AD and between the same || lines AD and BQ.

$\therefore$ ar. $(\Delta ADQ) = \frac{1}{2}$ ar. (|| gm ABCD) ...(ii)

Adding eqn. (i) and (ii), we get

ar. (ΔAPB) + ar. $(\Delta ADQ) = \frac{1}{2}$ ar. (|| gm ABCD) $+ \frac{1}{2}$ (||gm ABCD)

$\Rightarrow$ ar. (quad. ADQB) − ar. (ΔBPQ) = ar. (||gm ABCD)

$\Rightarrow$ ar. (quadrilateral ADQB) − ar. (ΔBPQ)

= ar. (quadrilateral ADQB − ar. (ΔDCQ)

$\Rightarrow$ ar. (ΔBPQ) = ar. (ΔDCQ)

Subtracting ar. (ΔPCQ) from both sides, we get

ar. (ΔBPQ) − ar. (ΔPCQ) = ar. (ΔDCQ) − ar. (ΔPCQ)

ar. (ΔBCP) = ar. (ΔDPQ).

Q.50. In the adjoining figure, ABCD is a parallelogram and O is any point on its diagonal AC. Show that : ar. (ΔAOB) = ar. (ΔAOD).

Ans. In ||gm ABCD, O is any point on its diagonal OB and OD are joined.

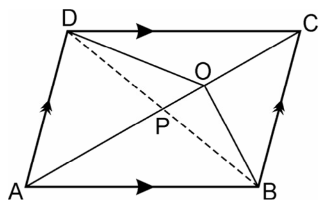
To Prove : ar. (ΔAOB) = ar. (ΔAOD)

Construction : Join BD which intersects AC at P.

Proof : In parallelogram ABCD diagonals of a || gm bisect each other.

$\therefore$ AP = PC and BP = PD

In ΔABD, AP is its median



$\therefore$ ar. (ΔABP) = ar. (ΔADP) ...(i)

Similarly in ΔOBD, OP is the median

$\therefore$ ar. (ΔOBP) = ar. (ΔODP) ...(ii)

Adding (i) and (ii), we get

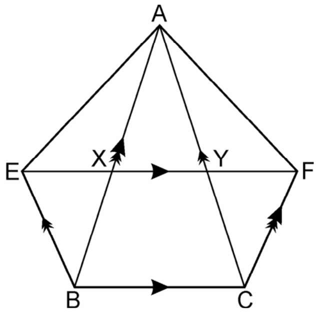
ar. (ΔAPB) + ar. (ΔOBP) = ar. (ΔADP) + ar. (ΔODP)

$\Rightarrow$ ar. (ΔAOB) = ar. (ΔAOD).

Q.51. In the given figure, XY || BC, BE || AC and CF || AB. Prove that : ar. (ΔABE) = ar. (ΔACF)

Ans. Given : In the figure, XY || BC, BE || AC and CF || AB.

To Prove : ar. (ΔABE) = ar. (ΔACF)



Proof : ΔABE and || gm BCYE are on the same base BE and between the same parallels

$\therefore$ ar. (ΔABE) = $\frac{1}{2}$ ar. (|| gm BCYE) ...(i)

Similarly ΔACF and || gm BCFX are on the same base CF and between the same parallels AB || CF.

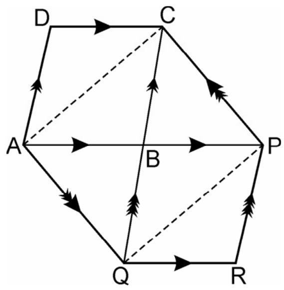
$\therefore$ ar. (ΔACF) = $\frac{1}{2}$ ar. (|| gm BCFX) ...(ii)

But || gm BCFX and || gm BCYE are on the same base BC and between the same parallels.

$\therefore$ ar. (||gm BCFX) = ar. (||gm BCYE) ...(iii)

From eqn. (i), (ii) and (iii), we get

ar. (ΔABE) = ar. (ΔACF).



Q.52. In the given figure, the side AB of || gm ABCD is produced to a point P. A line through A drawn parallel to CP meets CB produced in Q and the parallelogram PBQR is completed. Prove that :

ar. (|| gm ABCD) = ar. (|| gm BPRQ).

Ans. Given : Side AB of || gm ABCD is produced to P. CP is joined, through A, a line is drawn parallel to CP meeting CB produced at Q and || gm PBQR is completed as shown in the figure.

To Prove : ar. (||gm ABCD) = ar. (|| gm BPRQ)

Construction : Join AC and PQ.

Proof : In parallelogram ABCD, ΔAQC and ΔAQP are on the same base AQ and between the same parallels, then ar. (ΔAQC) = ar. (ΔAQP)

Subtracting ar. (ΔAQB) from both sides,

ar. (ΔAQC) – ar. (ΔAQB) = ar. (ΔAQP) – ar. (ΔAQB)

$\Rightarrow$ ar. (ΔABC) = ar. (ΔBPQ) ...(i)

But ar. (ΔABC) = $\frac{1}{2}$ ar. (|| gm ABCD) ...(ii)

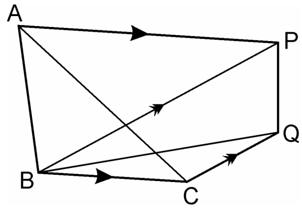
and ar. (ΔBPQ) = $\frac{1}{2}$ ar (|| gm BPRQ) ...(iii)

From (i), (ii) and (iii), we get

$$= \frac{1}{2} \text{ar. (|| gm ABCD)} = \frac{1}{2} \text{ar. (|| gm BPRQ)}$$

$\Rightarrow$ ar. (|| gm ABCD) = ar. (|| gm BPRQ).

Q.53. In the given figure, AP is parallel to BC, BP is parallel to CQ. Prove that the areas of triangles ABC and BQP are equal.



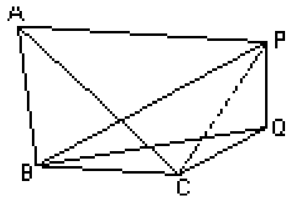
Ans. Given : AP || BC and BP || CQ.

To Prove : ar. (ΔABC) = ar. (ΔBPQ)

Construction : Join PC.

Proof : ΔABC and ΔBPC are on the same base BC and between the same || lines AP and BC.

$\therefore$ ar. (ΔABC) = ar. (ΔBPC) ...(i)



$\therefore$ ΔBPC and ΔBQP are on the same base BP and between the same || lines, BP and CQ.

$\therefore$ ar. (ΔBPC) = ar. (ΔBQP) ...(ii)

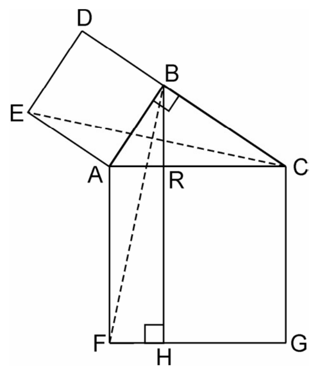
From (i) and (ii), we get

ar. (ΔABC) = ar. (ΔBQP)

Q.54. In the figure given along side squares ABDE and AFGC are drawn on the side AB and the hypotenuse AC of the right triangle ABC. If BH is perpendicular to FG, prove that :

(i) $\Delta EAC \cong \Delta BAF$

(ii) Area of square ABDE = Area of rectangle ARHF.



Ans. Given : A right angled ΔABC in which $\angle B = 90°$. Square ABDE and AFGC are drawn on side AB and hypotenuse AC of ΔABC. EC and BF are joined. $BH \perp FG$ meeting AC at R.

To Prove : (i) $\Delta EAC \cong \Delta BAF$

(ii) ar. (square ABDE) = ar. (rectangle ABHF)

Proof : (i) $\angle EAC = \angle EAB + \angle BAC$

$\Rightarrow$ $\angle EAC = 90° + \angle BAC$ ...(i)

$\angle BAF = \angle FAC + \angle BAC$

$\Rightarrow$ $\angle BAF = 90° + \angle BAC$ ...(ii)

From (i) and (ii), we get

$\angle EAC = \angle BAF$

In ΔEAC and ΔBAF, we have, EA = AB

$\angle EAC = \angle BAF$ and AC = AF

$\therefore$ $\Delta EAC \cong \Delta BAF$ (SAS axiom of congruency)

(ii) $\Delta EAC \cong \Delta BAF$ [Proved in part (i) above]

$\therefore$ ar. $(\Delta EAC) =$ ar. (ΔBAF)

$\angle ABD + \angle ABC = 90° + 90° \Rightarrow \angle ABD + \angle ABC = 180°$

$\therefore$ DBC is a straight line.

Now, ΔEAC and square ABDE are on the same base AE and between the same ‖ lines AF and BH.

$\therefore$ ar. $(\Delta EAC) = \frac{1}{2}$ ar. (square ABDE) ...(ii)

Again, ΔBAF and rectangle ARHF are on the same base AF and between the same ‖ lines AF and BH.

$\therefore$ ar. $(\Delta BAF) = \frac{1}{2}$ ar. (rectangle ARHF) ...(iii)

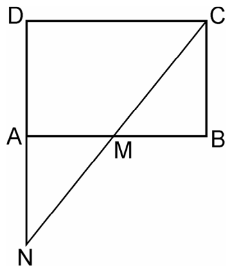
Since, ar. $(\Delta EAC) =$ ar. (ΔBAF)

From (ii) and (iii), we get

$\frac{1}{2}$ ar. (square ABDE) $= \frac{1}{2}$ ar. (rectangle ARHF)

$\Rightarrow$ ar. (square ABDE) = ar. (rectangle ARHF)

Q.55. M is the mid-point of side AB of rectangle ABCD. CM is produced to meet DA produced at point N. Prove that the parallelogram ABCD and triangle CDN are equal in area.



Ans. Given : M is mid-point of side AB of rectangle ABCD.

CM is joined and produced to meet DA produced at N.

To Prove : ar. (ABCD) = ar. (ΔCDN)

Proof : In ΔAMN and ΔBMC.

$\angle AMN = \angle BMC$ (Vertically opposite angles)

AM = MB ($\because$ M is mid-point of AB)

$\angle A = \angle B = 90°$

$\therefore$ $\Delta AMN \cong \Delta BMC$ (ASA axiom of congruency)

$\therefore$ ar. $(\Delta AMN) =$ ar. (ΔBMC)

Adding area of quad. AMCD both sides,

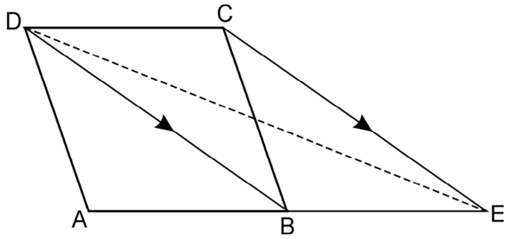
ar. $(\Delta AMN) +$ ar. (quad. AMCD)

$=$ ar. $(\Delta BMC) +$ ar. (quad. AMCD)

$\Rightarrow$ ar. $(\Delta CDN) =$ ar. (rectangle ABCD).

Q.56. In the adjoining figure, CE is drawn parallel to DB to meet AB produced at E. Prove that : ar. (quad. ABCD) = ar. (ΔDAE).

Ans. Given : In the given figure, CE is drawn parallel to BD which meets AB produced at E. DE is joined.



To Prove :

ar. (quad. ABCD) = ar. (ΔDAE)

Proof : ΔDBE and ΔDBC are on the same base BD and between the same parallels.

∴ ar. (ΔDBE) = ar. (ΔDBC)

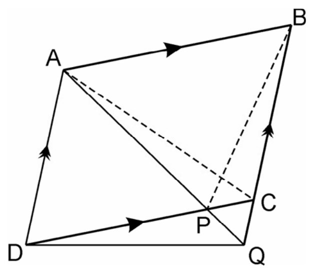
Adding ar. (ΔABD) both sides,

ar. (ΔDBE) + ar. (ΔABD) = ar. (ΔDBC) + ar. (ΔABD)

⇒ ar. (ΔADE) = ar. (quad ABCD)

Hence, ar. (quad ABCD) = ar. (ΔDAE).

Q.57. In the adjoining figure, ABCD is a parallelogram. Any line through A cuts DC at a point P and BC produced at Q. Prove that : ar. (ΔBPC) = ar. (ΔDPQ).



Ans. Given : ABCD is a || gm. A line through A, intersects DC at a point P and BC produced at Q.

To Prove : ar. (ΔBPC) = ar. (ΔDPQ)

Construction : Join AC and BP.

Proof : ΔBPC and ΔAPC are on the same base PC and between the same parallels.

∴ ar. (ΔBPC) = ar. (ΔAPC) ...(i)

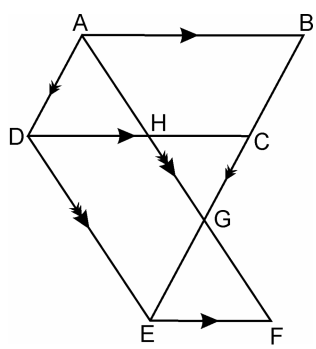
Again ΔAQC and ΔDQC and on the same base QC and between the same parallels.

∴ ar. (ΔAQC) = ar. (ΔDQC) ...(ii)

ar. (ΔBPC) = ar. (ΔAPC) = ar. (ΔAQC) – ar. (ΔPQC)

= ar. (ΔDQC) – ar. (ΔPQC) = ar. (ΔDPQ).

Q.58. In the given figure, AB || DC || EF, AD || BE and DE || AF. Prove that : ar. (|| gm DEFH) = ar. (|| gm ABCD).



Ans. Given : From figure, AB || DC || EF, AD || BE and DE || AF.

To Prove : ar. (|| gm DEFH) = ar. (|| gm ABCD)

Proof : In || gm ABCD and || gm ADEG are on the same base AD and between the same parallels.

∴ ar. (|| gm ABCD) = ar. (|| gm ADGE) ...(i)

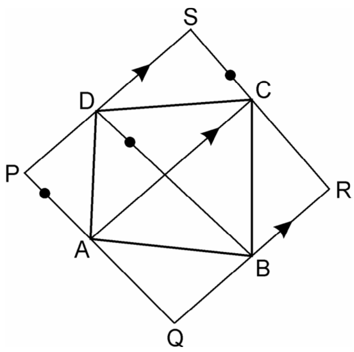
Similarly || gm DEFH and || gm ADEG are on the same base DE and between the same parallels.

$\therefore$ ar. (|| gm DEFH) = ar. (|| gm ADGE) ...(ii)

From (i) and (ii), we get

ar. (|| gm ABCD) = ar. (|| gm DEFH).

Q.59. In the following figure, AC || PS || QR and PQ || DB || SR. Prove that area of quadrilateral PQRS $= 2 \times$ ar. (quad ABCD).



Ans. Given : In the figure, ABCD and PQRS are two quadrilaterals such that AC || PS || QR and PQ || DB || SR.

To Prove : ar. (quad. PQRS) $= 2\times$ ar. (quad. ABCD)

Proof : In || gm PQRS, AC || PS || QR and PQ || DB || SR.

Similarly AQRC and APSC are also || gms.

$\because$ ΔABC and || gm AQRC are on the same base AC and between the same parallels, then

$\therefore$ ar. (ΔABC) $= \frac{1}{2}$ ar. (AQRC) ...(i)

Similarly, ar. (ΔADC) $= \frac{1}{2}$ ar. (APSC) ...(ii)

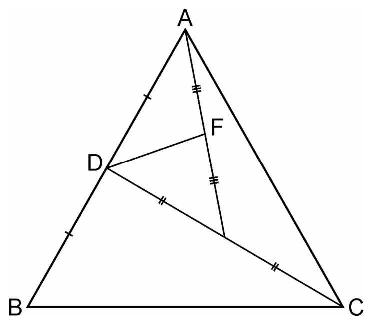
Adding (i) and (ii), we get

$\Rightarrow$ ar. (ΔABC) + ar. (ΔADC) $= \frac{1}{2}$ ar. (AQRC) $+ \frac{1}{2}$ ar. (APSC)

ar. (quad. ABCD) $= \frac{1}{2}$ ar. (quad. PQRS)

$\Rightarrow$ ar. (quad. PQRS) = 2 ar. (quad. ABCD).

Q.60. D is the mid-point of side AB of the triangle ABC, E is mid-point of CD and F is mid-point of AE. Prove that : $8\times$ ar. (ΔAFD) = ar. (ΔABC).



Ans. Given : ΔABC in which D is the mid-point of AB; E is the mid-point of CD and F is the mid-point of AE.

To Prove : $8\times$ ar. (ΔAFD) = ar. (ΔABC)

Proof : In ΔABC, D is mid-point of AB (Given)

$\therefore$ CD is the median of AB

ar. (ΔADC) $= \frac{1}{2}$ ar. (ΔABC)

$\Rightarrow$ 2ar. (ΔADC) = ar. (ΔABC) ...(i)

E is the mid-point of CD (Given)

$\therefore$ AE is the median of CD in ΔADC

$\therefore$ ar. $(\Delta ADE) = \frac{1}{2}$ ar. $(\Delta ADC) \Rightarrow 2$ ar. $(\Delta ADE) = \frac{1}{2}$ ar. (ΔADC)

$\Rightarrow$ ar. $(\Delta ADC) = 2$ ar. (ΔADE) ...(ii)

From (i) and (ii), we get

2×2 ar. $(\Delta ADE) =$ ar. (ΔABC)

$\Rightarrow$ 4 ar. $(\Delta ADE) =$ ar. (ΔABC) ...(iii)

F is the mid-point of AE, $\therefore$ 2 ar. $(\Delta AFD) =$ ar. (ΔADE)

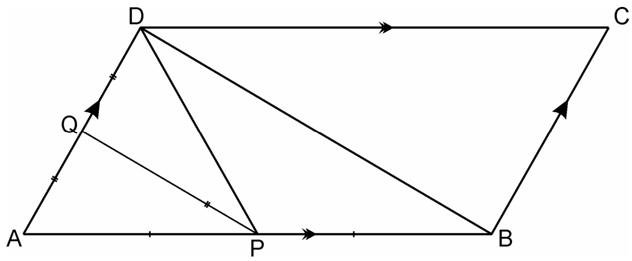
$\Rightarrow$ ar. $(\Delta ADE) = 2$ ar. (ΔADF) ...(iv)

From (iii) and (iv), we get

4×2 ar. $(\Delta AFD) =$ ar. (ΔABC)

Hence, $8 \times$ ar. $(\Delta AFD) =$ ar. (ΔABC)

Q.61. ABCD is a parallelogram. P and Q are the mid-points of sides AB and AD respectively. Prove that area of triangle APQ $= \frac{1}{8}$ of the area of parallelogram ABCD.



Ans. Given : || gm ABCD in which P is the mid-point of AB and Q is the mid-point of AD. PQ is joined.

To Prove : ar. $(\Delta APQ) = \frac{1}{8}$ ar. (|| gm ABCD)

Construction : Join PD and BD.

Proof : In parallelogram ABCD, diagonal of a || gm divides it into two equal parts. Since, BD is diagonal, then ar. (|| gm ABCD) = 2 ar. (ΔABD) ...(i)

In ΔABD, DP is the median of AB.

$\therefore$ ar. $(\Delta ABD) = 2$ ar. (ΔADP) ...(ii)

From (i) and (ii), we get, ar. (|| gm ABCD) = 2 [2 ar. (ΔADP)]

$\Rightarrow$ ar. (|| gm ABCD) = 4 ar. (ΔADP) ...(iii)

In ΔADP, PQ is median of AD.

$\therefore$ ar. $(\Delta ADP) = 2$ ar. (ΔAQP) ...(iv)

From eqn. (iii) and (iv), we get

ar. (|| gm ABCD) = 4×2 ar. $(\Delta APQ) \Rightarrow$ ar. (|| gm ABCD) = 8 ar. (ΔAPQ)

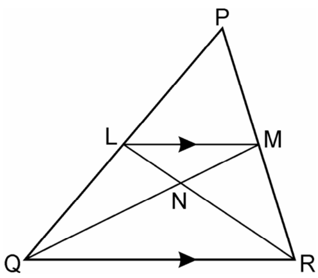
Hence, ar. (ΔAPQ) $= \frac{1}{8}$ ar. (|| gm ABCD).

Q.62. In the given triangle PQR, LM is parallel to QR and PL : LQ = 3 : 4. Calculate the value of ratio :

(i) $\frac{PL}{PQ}, \frac{PM}{PR}$ **and** $\frac{LM}{QR}$

(ii) $\frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR}$

(iii) $\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta LQN}$



Ans. (ii) In ΔPQR, L is mid-point of PQ and M is mid-point of PR,

$$\frac{PL}{LQ} = \frac{3}{4} \Rightarrow \frac{PL}{PL+LQ} = \frac{3}{3+4} \Rightarrow \frac{PL}{PQ} = \frac{3}{7}$$

LM || QR in ΔPQR (Given)

$\therefore \quad \frac{PM}{PR} = \frac{PL}{PQ} = \frac{3}{7} \qquad \therefore \frac{PM}{PR} = \frac{3}{7}$

Again, $\frac{PM}{PR} = \frac{PL}{PQ} = \frac{LM}{QR} = \frac{3}{7}$

Thus, $\frac{LM}{QR} = \frac{3}{7}$

(ii) $\frac{\text{ar. }(\Delta LMN)}{\text{ar. }(\Delta MNR)} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$ [$\because$ Δs LMN and QNR]

(iii) $\frac{\text{ar. }(\Delta LQM)}{\text{ar. }(\Delta LQN)} = \frac{LM}{QN} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$.