Q.1. Find the total surface area and volume of a rectangular solid (cuboid) measuring 1 m by 50 cm by 0.5 m.

Ans. Length of cuboid \( l = 1 \) m, Breadth of cuboid, \( b = 50 \) cm = \( \frac{50}{100} = \frac{1}{2} \) m  

Height of cuboid, \( h = 0.5 \) m = \( \frac{5}{10} \) m = \( \frac{1}{2} \) m

\[ \therefore \text{Volume of cuboid} = l \times b \times h = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ m}^3 = 0.25 \text{ m}^3. \]

Total surface area of cuboid = \( 2 (lb + bh + hl) = 2 \left( \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1 \right) \)

\[ = 2 \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) = 2 \left( \frac{2+1+2}{4} \right) \]

\[ = 2 \times \frac{5}{4} = \frac{5}{2} = 2.50 \text{ m}^2 \]

Hence, volume of cuboid is 0.25 m\(^3\) and total surface area of cuboid is 2.50 m\(^2\).

Q.2. The length, breadth and height of a rectangular solid are in the ratio 5 : 4 : 2. If the total surface area is 1216 cm\(^2\), find the length, breadth and height of the solid.

Ans. Let length, \( l = 5x \), breadth, \( b = 4x \) and height, \( h = 2x \).

\[ \therefore \text{Total surface area} = 1216 \text{ cm}^2 \]

\[ \Rightarrow 2 (lb + bh + hl) = 1216 \]

\[ \Rightarrow 2 (5x \times 4x + 4x \times 2x + 2x \times 5x) = 1216 \]

\[ \Rightarrow 2 (20x^2 + 8x^2 + 10x^2) = 1216 \Rightarrow 2 (38x^2) = 1216 \]

\[ \Rightarrow 76x^2 = 1216 \Rightarrow x^2 = \frac{1216}{76} = 16 \Rightarrow x^2 = 16 \Rightarrow x = 4 \therefore l = 5x = 5 \times 4 = 20 \text{ cm} \]

\[ b = 4x = 4 \times 4 = 16 \text{ cm}, \text{ and } h = 2x = 2 \times 4 = 8 \text{ cm} \]

Hence, length, \( l = 20 \) cm, breadth, \( b = 16 \) cm, and height \( h = 8 \) cm.
Q.3. The volume of a rectangular wall is 33 $m^3$. If its length is 16.5 m and height 8 m, find the width of the wall.

Ans. Volume of rectangular wall = 33 $m^2$
Length of wall ($l$) = 16.5 m, Height of wall ($h$) = 8 m
Let width of wall = $b$ m, then volume of rectangular wall = $l \times b \times h$

$\Rightarrow 16.5 \times 8 \times b = 33$

$\Rightarrow b = \frac{33}{16.5 \times 8} = \frac{1}{0.25} = 4$ m

Hence, width of wall is 0.25 m.

Q.4. A class room is 12.5 m long, 6.4 m broad and 5 m high. How many students can accommodate if each student needs 1.6 $m^2$ of floor area? How many cubic metres of air would each student get?

Ans. Length of room ($l$) = 12.5 m, width of room ($b$) = 6.4 m and height of room ($h$) = 5 m

∴ Volume of air inside the room = $l \times b \times h = 12.5 \times 6.4 \times 5 = 400$ m$^3$

Area of floor of the room = $l \times b = 12.5 \times 6.4 = 80$ m$^2$

For each student area required = 1.6 m$^2$

∴ Number of students = $\frac{80}{1.6} = \frac{80 \times 10}{16} = 50$

The required air received by each student = $\frac{400}{50} = 8$ m$^3$

Q.5. Find the length of the longest rod that can be placed in a room measuring 12 m $\times$ 9 m $\times$ 8 m.

Ans. Length of room ($l$) = 12 m, breadth of room ($b$) = 9 m, and height of room ($h$) = 8 m

Hence, the longest rod required to place in the room

$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(12)^2 + (9)^2 + (8)^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$ m

Q.6. The volume of a cuboid is 14400 $cm^3$ and its height is 15 cm. The cross-section of the cuboid is a rectangle having its sides in the ratio 5 : 3. Find the perimeter of the cross-section.

Ans. Volume of cuboid = 14400 $cm^3$, and Height of cuboid ($h$) = 15 cm

∴ Length $\times$ Breadth = $\frac{volume}{h} = \frac{14400}{15} = 960$ $cm^2$

Ratio of remaining sides = 5 : 3
Thus, length = $5x$ and breadth = $3x$
\[ 5x \times 3x = 960 \Rightarrow 15x^2 = 960 \Rightarrow x^2 = 64 = (8)^2 \Rightarrow x = 8 \]
Length = 5x = 8 \times 8 = 40 \text{ cm} \quad \text{and breadth} = 3x = 8 \times 3 = 24 \text{ cm} 
Hence, perimeter of rectangular cross-section = 2(l + b) = 2(40 + 24) \text{ cm} 
\[ = 2 \times 64 = 128 \text{ cm}. \]

**Q.7. The area of path is 6500 m}^2. Find the cost of covering it with gravel 14 cm deep at the rate of Rs 5.60 per cubic metre.**

**Ans.** Area of path = 6500 m}^2 \quad \text{and Depth of gravel} = 14 \text{ cm} = \frac{14}{100} \text{ m} 
\[ \therefore \quad \text{Volume of gravel} = \text{Area} \times \text{Depth} = 6500 \times \frac{14}{100} = 910 \text{ m}^3 \]
Rate of covering the gravel = Rs 5.60 per m}^3 
\[ \therefore \quad \text{Total cost} = \text{Rs} \ 910 \times 5.60 = \text{Rs} \left( \frac{910 \times 560}{100} \right) = \text{Rs} \ 5096. \]

**Q.8. The cost of papering the four walls of a room 12 m long at Rs 6.50 per square metre is Rs 1638 and the cost of matting the floor at Rs 3.50 per square metre is Rs 378. Find the height of the room.**

**Ans.** Rate of papering the walls = Rs 6.50 per m}^2 \quad \text{and Total cost} = \text{Rs} \ 1638 
\[ \therefore \quad \text{Area of four walls} = \frac{1638}{6.50} = \frac{1638 \times 100}{650} \text{ m}^2 = 252 \text{ m}^2 \]
Rate of matting the floor = Rs 3.50 per m}^2 \quad \text{and Total cost} = \text{Rs} \ 378 
\[ \therefore \quad \text{Area of floor} = \frac{378}{3.50} = \frac{378 \times 100}{350} \text{ m}^2 = 108 \text{ m}^2 \]
Length of room = 12 m \quad \therefore \quad \text{Breadth of room} = \frac{\text{Area of floor}}{\text{Length}} = \frac{108}{12} = 9 \text{ m} 
\[ \text{Area of four walls} = 2(l + b)h = 252 \]
\[ \Rightarrow \quad 2(12 + 9)h = 252 \Rightarrow 2 \times 21h = 252 \Rightarrow 21h = 252 \]
\[ \Rightarrow \quad h = \frac{252}{21} \Rightarrow h = 6 \]
Hence, height of the room is 6 m.
Q.9. The dimensions of a field are 15 m × 12 m. A pit 7.5 m × 6 m × 1.5 m is dug in one corner of the field and the earth removed from it, is evenly spread over the remaining area of the field, calculate, by how much the level of the field is raised?

**Ans.** Length of field \((l) = 15\) m and breadth of field \((b) = 12\) m

\[ \therefore \text{Area of total field} = l \times b = 15 \times 12 = 180 \text{ m}^2 \]

Length of pit = 7.5 m breadth of pit = 6 m and depth of pit = 1.5 m

\[ \therefore \text{Area of pit} = l \times b = 7.5 \times 6 = 45 \text{ m}^2 \]

and volume of earth removed = \(l \times b \times h = 7.5 \times 6 \times 1.5 \text{ m}^3 = 67.5 \text{ m}^3 \)

\[ \therefore \text{Area of field leaving pit } = (180 - 45) \text{ m}^2 = 135 \text{ m}^2 \]

and volume of earth removed = \(l \times b \times h = 7.5 \times 6 \times 1.5 \text{ m}^3 = 67.5 \text{ m}^3 \)

Area of field leaving pit = \(180 - 45\) m² = 135 m²

\[ \therefore \text{Level (height) of the earth in the field} = \frac{\text{Volume of earth}}{\text{Area of remaining part}} = \frac{67.5}{135} = \frac{675}{10 \times 135} \text{ m} = 0.5 \text{ m} = 50 \text{ cm} \]

Q.10. The sum of length, breadth and depth of a cuboid is 19 cm, and the length of its diagonal is 11 cm. Find the surface area of the cuboid.

**Ans.** Let \(l, b\) and \(h\) be the length, breadth and depth of the cuboid, then

\[ l + b + h = 19 \text{ cm} \]

and \(\sqrt{l^2 + b^2 + h^2} = 11 \text{ cm} \Rightarrow l^2 + b^2 + h^2 = (11)^2 = 121 \)

The surface area of the cuboid = \(2(lb + bh + hl)\)

We know that \((l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)\)

\[ \Rightarrow (19)^2 = 121 + 2(lb + bh + hl) \Rightarrow 2(lb + bh + hl) = (19)^2 - 121 \]

\[ = 361 - 121 = 240 \text{ cm}^2 \]

Hence, surface area of the cuboid = 240 cm²

Q.11. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 50 cm thick, while the cement and the sand mixture occupies \(\frac{1}{20}\) th of the volume of the wall.

**Ans.** Volume of one brick = \(25 \times 12.5 \times 7.5 \text{ cm}^3 = \frac{25}{100} \times \frac{12.5}{100} \times \frac{7.5}{100} \text{ m}^3 = \frac{1}{4} \times \frac{1}{8} \times \frac{3}{40} \text{ m}^3 = \frac{3}{1280} \text{ m}^3\)
Length of wall \((l) = 6\) m and Height of wall \((h) = 5\) m

and thickness of wall \((b) = \frac{50}{100} = \frac{1}{2}\) m

\[ \therefore \text{Volume of wall} = l \times b \times h = 6 \times \frac{1}{2} \times 5 = 15 \text{ m}^3 \]

Volume of cement and sand \(= \frac{1}{20} \) of \(15 \text{ m}^3 = \frac{3}{4} \text{ m}^3 \)

\[ \therefore \text{Volume of bricks} = \text{Volume of wall} - \text{Volume of cement and sand} \]

\[ = \left( \frac{60 - 3}{4} \right) \text{m}^3 = \frac{57}{4} \text{ m}^3 \]

\[ \therefore \text{Number of bricks} = \frac{\text{Volume of total bricks}}{\text{Volume of one brick}} \]

\[ = \frac{4}{3} \times \frac{57}{3} \times \frac{1280}{4} = 19 \times 320 = 6080 \]

Q.12. The total surface area of a cube is 726 cm\(^2\). Find its volume.

\textbf{Ans.} Total surface area of cube = 726 cm\(^2\)

Let edge of cube = \(a\) cm

\[ \therefore \text{Total surface area of cube} = 6a^2 = 726 \]

\[ \Rightarrow a^2 = \frac{726}{6} = 121 \Rightarrow a = \sqrt{121} = 11 \text{ cm} \]

\[ \therefore \text{Edge of cube} = 11 \text{ cm} \]

Volume of cube = (side\(^3\) = (11 cm\(^3\) = 11\times11\times11 \text{ cm}^3 = 1331 \text{ cm}^3 \)

Q.13. The volume of a cube is 729 cm\(^3\). Find its total surface area.

\textbf{Ans.} Volume of cube = 729 cm\(^3\)

Let edge of cube = \(a\)

\[ \therefore a^3 = 729 = 9\times9\times9 \therefore a = 9 \text{ cm} \]

\[ \therefore \text{Total surface area of cube} = 6a^2 = 6\times9 \text{ cm}\times9 \text{ cm} = 486 \text{ cm}^2 \]

Q.14. The edges of three cubes of metal are 3 cm, 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.

\textbf{Ans.} The edges of three cubes are 3 cm, 4 cm and 5 cm.

Volume of three cubes are \((3)^3\), \((4)^3\) and \((5)^3\) i.e. \(27 \text{ cm}^3\), \(64 \text{ cm}^3\) and \(125 \text{ cm}^3\).

\[ \therefore \text{Volume of new cube} = 27 + 64 +125 = 216 \text{ cm}^3 \]

Let edge of new cube be \(a\)
\[ a^3 = 216 = 6 \times 6 \times 6, \quad a = 6 \text{ cm} \]

Hence, edge of new cube = 6 cm

Q.15. Three cubes, whose edges are \( x \) cm, 8 cm and 10 cm respectively, are melted and recasted into a single cube of edge 12 cm. Find ‘\( x \)’.

Ans. Edges of three cubes are \( x \) cm, 8 cm and 10 cm respectively

\[ \therefore \text{Volume of these cubes are } x^3, (8)^3 \text{ and } (10)^3 \text{ i.e. } x^3 \text{ cm}^3, 512 \text{ cm}^3 \text{ and } 1000 \text{ cm}^3 \]

Edge of new cube formed = 12 cm

\[ \therefore \text{Volume of new cube } = (\text{side})^3 = (12)^3 = 12 \times 12 \times 12 = 1728 \text{ cm}^3 \]

According to the given problem

\[ x^3 + 512 + 1000 = 1728 \Rightarrow x^3 + 1512 = 1728 \]

\[ \Rightarrow x^3 = 1728 - 1512 = 216 \Rightarrow x = 6 \text{ cm} \]

Hence, edge of cube = \( x = 6 \) cm.

Q.16. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.

Ans. Let edge of each of three cubes be ‘\( x \)’ when three cubes are placed end to end, a cuboid is formed whose dimensions are:

\[ l = x + x + x = 3x, \quad b = x, \quad h = x \]

\[ \therefore \text{Total surface area of a resulting cuboid } = 2 (lb + bh + hl) \]

\[ = 2 (3\times x \times x + x \times x + x \times 3x) \]

\[ = 2 (3x^2 + x^2 + 3x^2) = 2 \times 7x^2 = 14x^2 \]

Sum of total surface areas of three cubes \[ = 3 \times 6x^2 = 18x^2 \]

\[ \therefore \text{Ratio of surface areas of cubes } = \frac{14x^2}{18x^2} = \frac{14}{18} = \frac{7}{9} \]

Hence, the required ratio is 7 : 9.

Q.17. A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of two smaller cubes are 6 cm and 8 cm, find the edge of third smaller cube.

Ans. Edge of metal cube = 12 cm

\[ \therefore \text{Volume of metal cube } = (12)^3 = 1728 \text{ cm}^3 \]

Edge of first smaller cube = 6 cm

\[ \therefore \text{Volume of smaller cube } = (6)^3 = 216 \text{ cm}^3 \]

Edge of second smaller cube = 8 cm
Q.18. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find (i) the edge of the cube (ii) the surface area of the cube.

Ans. Length of the metallic cuboid \( l = 100 \) cm

breadth of the metallic cuboid \( b = 80 \) cm

and height of metallic cuboid \( h = 64 \) cm

\[ \text{Volume of metallic cuboid} = l \times b \times h = 100 \times 80 \times 64 = 512000 \text{ cm}^3 \]

\[ \text{Volume of cube so formed} = 512000 \text{ cm}^3 \]

(i) Edge of the cube \( a = (512000)^{1/3} = [(80)^3]^{1/3} = 80 \) cm

(ii) Surface area of cube \( = 6a^2 = 6(80)^2 = 6 \times 80 \times 80 \text{ cm}^2 = 38400 \text{ cm}^2 \)

Q.19. The square on the diagonal of cube has an area of 1875 cm\(^2\), Calculate (i) the side of the cube (ii) the total surface area of the cube.

Ans. \((\text{Diagonal})^2 \) of cube \( = 1875 \text{ cm}^2 \)

(i) Let side of cube \( = a \)

\[ (\sqrt{3}a)^2 = 1875 \Rightarrow 3a^2 = 1875 \Rightarrow a^2 = \frac{1875}{3} \]

\[ a^2 = 625 = (25)^2 \Rightarrow a = 25 \]

Hence, side of the cube is 25 cm.

(ii) Total surface area of the cube = 6 side\(^2\) = 6\(a^2\) = 6\((25)^2\) = 6\(\times\)625 \text{ cm}^2 = 3750 \text{ cm}^2

Q.20. Four identical cubes are joined end to end to form a cuboid. If the total surface area of the resulting cuboid is 648 cm\(^2\). Find the length of edge of each cube. Also find the ratio between the surface area of resulting cuboid and the surface area of a cube.

Ans. Surface area of cuboid \( = 648 \text{ cm}^2 \), let edge of each cube \( = x \) cm

\[ \text{Length of cuboid} = 4x \text{ cm}, \text{width of cuboid} = x \text{ cm} \text{ and height of cuboid} = x \text{ cm} \]

\[ \text{Surface area of cuboid} = 2(lb + bh + hl) \]

\[ 648 = 2(4x \times x + x \times x + x \times 4x) = 2(4x^2 + x^2 + 4x^2) \]

\[ 648 = 18x^2 \Rightarrow x^2 = \frac{648}{18} = 36 = (6)^2 \Rightarrow x = 6 \]
Thus, length of edge of each cube = 6 cm
Surface area of cube = 6 (side)$^2$ = $6 \times 6 = 6 \times (6)^2 = 6 \times 36 = 216$ cm$^2$
Hence, ratio between the surface area of cuboid and that of cube = $648 : 216 = 3 : 1$

Q.21. A rectangular container has base of length 12 cm and width 9 cm. A cube of edge 6 cm is placed in the container and then sufficient water is filled into it, so that the cube is just submerged. Find the fall in level of water in the container, when the cube is removed.

Ans. Length of the base of container = 12 cm and width of container = 9 cm
Let fall in water level be $x$ cm. \(\therefore\) Side of cube = 6 cm
\(\therefore\) Volume of cube = $6 \times 6 \times 6 = 216$ cm$^3$ \(\therefore\) Volume of water = 216 cm$^3$
According to the given problem, we get
\[12 \times 9 \times x = 216 \Rightarrow x = \frac{216}{12 \times 9} = 2\]
Hence, fall in water level is 2 cm.

Q.22. A rectangular container whose base is a square of side 12 cm, contains sufficient water to submerge a rectangular solid 8 cm $\times$ 6 cm $\times$ 3 cm. Find the rise in level of water in the container when the solid is in it.

Ans. Let the rise in water level = $x$ cm
Length of solid cuboid = 8 cm,
width of solid cuboid = 6 cm
Height of solid cuboid = 3 cm
\(\therefore\) Volume of cuboid = $l \times b \times h$
\[= 8 \times 6 \times 3 = 144$ cm$^3$\n\(\therefore\) Volume of water = 144 cm$^3$
Side of the square base = 12 cm
\(\therefore\) Area of base = $12 \times 12 = 144$ cm$^2$
According to given problem, we get
\[144 \times x = 144 \Rightarrow x = 1\]
Hence, rise in water level is 1 cm.

Q.23. A field is 120 m long and 50 m broad. A tank 24 m long, 10 m broad and 6 m deep is dug anywhere in the field and the earth taken out of the tank is evenly spread over the remaining part of the field. Find the rise in level of the field.

Ans. Length of rectangular field = 120 m, Breadth of rectangular field = 50 m
Area of of the total field = $120 \times 50$ m$^2$ = 6000 m$^2$
Length of tank = 24 m, breadth of tank = 10 m and depth of tank = 6 m
\(\therefore\) Volume of earth dug out = $24 \times 10 \times 6$ m$^3$ = 1440 m$^3$
Area of surface of tank = \( l \times b = 24 \times 10 = 240 \text{ m}^2 \)

\( \therefore \) Area of remaining part of the field

= Area of total field – Surface area of tank

= 6000 – 240 = 5760 \text{ m}^2

Let height of rise of field = \( x \) m

\( \therefore 5760 \times x = 1440 \)

\( x = \frac{1440}{5760} = \frac{1}{4} = 0.25 \). Hence, rise in level of the field = 0.25 m or 25 cm.

Q.24. The following figure shows a solid of uniform cross-section. Find the volume of the solid. All measurements are in centimetres. Assume that all angles in the figure are right-angles.

Ans. The given figure can be divided into two cuboids of dimensions 6 cm, 4 cm, 3 cm and 9 cm respectively. Hence, volume of solid

\( = 6 \times 4 \times 3 + 4 \times 3 \times 9 = 72 + 108 = 180 \text{ cm}^3 \)

Q.25. A swimming pool is 40 m long and 15 m wide. Its shallow and deep ends are 1.5 m and 3 m deep respectively if the bottom of the pool slopes uniformly, find the amount of water in litres required to fill the pool.

Ans. The cross-section of the swimming pool is a trapezium with parallel sides 1.5 m and 3 m (i.e. depths of both ends) and 40 m (i.e. length of pool) as the distance between \( \parallel \) sides.

The width of pool i.e. 15 m can be taken as the length of cross-section.

Length of swimming pool \( (l) = 40 \text{ m} \)

Breadth of swimming pool \( (b) = 15 \text{ m} \)

Depth of its ends = 1.5 m and 3.5 m

\( \therefore \) Volume of water = \( \frac{1}{2} \{(1.5 + 3.0) \times 40\} \times 15 \text{ m}^3 \)

\[ = \frac{1}{2} \times 4.5 \times 40 \times 15 \text{ m}^3 \]

\[ = 1350 \text{ m}^3 \]

\( \therefore \) Capacity of water in litres = 1350 \times 1000 litres \((1 \text{ m}^3 = 1000 \text{ litres})\)

\[ = 1350000 \text{ litres} \]
Q.26. The cross-section of a piece of metal 2 m in length is shown in the adjoining figure. Calculate:

(i) The area of its cross-section.
(ii) The volume of piece of metal.
(iii) The weight of piece of metal to the nearest kg, if 1 cm\(^3\) of the metal weighs 6.5 g.

**Ans.** From C, draw CF \(\parallel\) AB

then CF = AB = 13 cm
AF = CB = 8 cm

\[\therefore EF = EA - FA = 12 - 8 = 4\text{ cm}\]

(i) Now area of figure ABCDE (cross-section)

\[= \text{ar. (Rectangle ABCF) + ar. (Trapezium FCDE)}\]

\[= 13 \text{ cm} \times 8 \text{ cm} + \frac{1}{2}(13+16) \times 4 \text{ cm}^2\]

\[= 104 + 32 = 136\text{ cm}^2\]

(ii) Length of the piece = 2 m = 200 cm

\[\therefore\text{Volume of the metal piece = Area} \times \text{Length}\]

\[= 162 \times 200 \text{ cm}^3\]

\[= 32400 \text{ cm}^3\]

\[= \frac{32400}{100 \times 100} = 3.24 \text{ m}^3\]

(iii) Weight of 1 cm\(^3\) metal = 6.5 g

\[\therefore\text{Total weight of the metal piece} = 32400 \times 6.5\text{ g}\]

\[= 210600\text{ g}\]

\[= 211\text{ kg (approx)}\]

Q.27. A square brass plate of side \(x\) cm is 1 mm thick and weighs 5.44 kg. If 1 cm\(^3\) of brass weighs 8.5 g, find the value of \(x\).

**Ans.** Side of square plate = \(x\) cm
Thickness of square plate = 1 mm
Total weight of square plate = 5.44 kg

Weight of 1 cm\(^3\) = 8.5 g

\[\therefore\text{Total volume of the plate} = \frac{5.44 \times 1000}{8.5} \text{ cm}^3\]

\[= 640\text{ cm}^3\]
∴ According to the given problem \( x \times x \times \frac{1}{10} = 640 \)

\[ x^2 = 6400 \]

\[ x = \sqrt{6400} = 80 \text{ cm} \]

**Q.28.** The area of cross-section of a rectangular pipe is \( 5.4 \text{ cm}^2 \) and water is pumped out of it at the rate of 27 kmph. Find, in litres, the volume of water which flows out of the pipe in 1 minute.

**Ans.** Area of cross-section of rectangular pipe = \( 5.4 \text{ cm}^2 \)

Speed of water pumped out = 27 kmph

Time = 1 minute

∴ Length of water flow \( = \frac{27}{60} \times 1000 \text{ m} = 450 \text{ m} \)

∴ Volume of water = Area \( \times \) Length

\[
= 450 \times \frac{5.4}{100 \times 100} \text{ m}^3
\]

\[
= \frac{450 \times 54}{100 \times 100} \text{ m}^3
\]

\[
= \frac{24300}{10000} \text{ m}^3
\]

\[
= 2.43 \text{ m}^3
\]

∴ Volume of water in litres \( = \frac{2.43}{1000} \times 1000 \) \( (1 \text{ m}^3 = 1000 \text{ litres}) \)

\[
= 243 \text{ litres}
\]

**Q.29.** The cross-section of a tunnel, perpendicular to its length is a trapezium ABCD in which \( AB = 8 \text{ m}, DC = 6 \text{ m} \) and \( AL = BM \). The height of the tunnel is 2.4 m and its length is 40 m.

Find :

(i) The cost of paving the floor of the tunnel at Rs 16 per \( \text{ m}^2 \).

(ii) The cost of painting the internal surface of the tunnel, excluding the floor at the rate of Rs 5 per \( \text{ m}^2 \).

**Ans.** The cross-section of a tunnel is of the trapezium shaped ABCD in which \( AB = 8 \text{ m}, DC = 6 \text{ m} \) and \( AL = BM \). The height is 2.4 m and its length is 40 m.

(i) Area of floor of tunnel \( = l \times b = 40 \times 8 = 320 \text{ m}^2 \)
Rate of cost of paving = Rs 16 per m²
Total cost = 320×16 = Rs 5120

(ii) \( AL = BM = \frac{8-6}{2} = \frac{2}{2} = 1 \text{ m} \)

\[ \therefore \Delta ADL, \]
\[ AD^2 = AL^2 + DL^2 \quad \text{(Using Pythagoras Theorem)} \]
\[ = (1)^2 + (2.4)^2 = 1 + 5.76 = 6.76 \]
\[ = (2.6)^2 \]
\[ \therefore AD = 2.6 \text{ m} \]

Perimeter of the cross-section of the tunnel = \((8 + 2.4 + 2.4 + 6) \text{ m} = 18.8 \text{ m} \)

Length = 40 m

\[ \therefore \text{Internal surface area of the tunnel (except floor)} \]
\[ = (18.8 \times 40 - 40 \times 8) \text{ m}^2 \]
\[ = (752 - 320) \text{ m}^2 \]
\[ = 432 \text{ m}^2 \]

Rate of painting = Rs 5 per m²
Hence, total cost of painting = Rs 5×432 = Rs 2160

Q.30. A stream, which flows at a uniform rate of 4 km/hr, is 10 metres wide and 1.2 m deep at a certain point. If its cross-section is rectangular is shape find, in litres, the volume of water that flows in a minute.

\[ \text{Ans. Speed of stream} = 4 \text{ km/hr} = 4 \times \frac{5}{18} \text{ m/s} = \frac{10}{9} \text{ m/s} \]

Area of cross-section of stream = 10×1.2 = 12 m²

\[ \therefore \text{Volume of water discharged in 1 second} = 12 \times \frac{10}{9} = \frac{40}{3} \text{ m}^3 \]

But 1 m³ = 1000 litres
\[ \therefore \frac{40}{3} \text{ m}^3 = \frac{40}{3} \times 1000 = \frac{40000}{3} \text{ litres} \]

\[ \therefore \text{Volume of water discharged in 1 minutes i.e. 60 sec.} \]
\[ = \frac{40000}{3} \times 60 = 800000 \text{ litres.} \]
Q.31. If 100.8 cubic metres of sand be thrown into a rectangular tank 14 m long and 5 m wide; find the rise in level of the water.

**Ans.** Length of rectangular tank = 14 m
Width of rectangular tank = 5 m
Let rise in level = \( x \) m

\[
\text{Volume} = 14 \times 5 \times x = 70x \text{ m}^3
\]

But volume = 100.8 m\(^3\) (Given)

\[
70x = 100.8
\]

\[
x = \frac{100.8}{70} \Rightarrow x = \frac{1008}{70} \times \frac{1}{10}
\]

\[
\Rightarrow x = \frac{144}{100} \Rightarrow x = 1.44 \text{ m}
\]

Q.32. A rectangular card-board sheet has length 32 cm and breadth 26 cm. Squares of each side 3 cm are cut from the corners of the sheet and the sides are folded to make a rectangular container. Find the capacity of the container formed.

**Ans.** Length of sheet = 32 cm
Breadth of sheet = 26 cm
Side of each square = 3 cm

\[
\text{Inner length} = 32 - 2 \times 3 = 32 - 6 = 26 \text{ cm}
\]

\[
\text{Inner breadth} = 26 - 2 \times 3 = 26 - 6 = 20 \text{ cm}
\]

By folding the sheet, the length of the container = 26 cm
Breadth of the container = 20 cm and height of container = 3 cm

\[
\text{Volume of the container} = l \times b \times h = 26 \text{ cm} \times 20 \text{ cm} \times 3 \text{ cm} = 1560 \text{ cm}^3
\]

Q.33. A swimming pool is 18 m long and 8 m wide its deep and shallow ends are 2 m and 1.2 m respectively find the capacity of the pool, assuming that the bottom of the pool slopes uniformly.

**Ans.** Length of pool = 18 m
Breadth of pool = 8 m
Height of one side = 2 m
Height on second side = 1.2 m

\[
\text{Volume of pool} = 18 \times 8 \times \frac{(2 + 1.2)}{2} \text{ m}^3
\]

\[
= \frac{18 \times 8 \times 3.2}{2} = 230.4 \text{ m}^3
\]
Q.34. A plot is 83 m long and 35 m broad. A tank 15 m long, 7 m broad and 4 m deep is dug anywhere in the plot and the earth so removed is evenly spread over the remaining part of the plot. Find the rise in level of the plot.

Ans. Length of plot = 83 m and width of plot = 35 m

\[ \text{Area of plot} = 83 \times 35 \, \text{m}^2 = 2905 \, \text{m}^2 \]

\[ \text{Area of tank} = l \times b = 15 \times 7 \, \text{m}^2 = 105 \, \text{m}^2 \]

\[ \text{Area of remaining plot} = (2905 - 105) \, \text{m}^2 = 2800 \, \text{m}^2 \]

\[ \text{Volume of earth dug out} = l \times b \times d = 15 \times 7 \times 4 \, \text{m}^3 = 420 \, \text{m}^3 \]

\[ \text{Height of level of earth spread over the remaining part} = \frac{\text{Volume of earth}}{\text{Area of plot}} \]

\[ = \frac{420}{2800} \, \text{m} = \frac{3}{20} \, \text{m} = \frac{3}{20} \times 100 = 15 \, \text{cm} \]

Q.35. Find the volume of the cylinder in which:

(i) Height = 21 cm and Base radius = 5 cm

(ii) Diameter = 28 cm and Height = 40 cm.

Ans. (i) Height of cylinder \( (h) = 21 \, \text{cm} \) and radius of the base \( (r) = 5 \, \text{cm} \)

\[ \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 21 \, \text{cm}^3 = 1650 \, \text{cm}^3 \]

(ii) Radius of the base of cylinder, \( r = \frac{28}{2} = 14 \, \text{cm} \)

Height of cylinder \( (h) = 40 \, \text{cm} \)

\[ \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 40 \, \text{cm}^3 \]

\[ = 22 \times 28 \times 40 = 24640 \, \text{cm}^3 \]

Q.36. Find the weight of the solid cylinder of radius 10.5 cm and height 60 cm, if the material of the cylinder weighs, 5 grams per cu. cm.

Ans. Radius of solid cylinder \( (r) = 10.5 \, \text{cm} \)

Height of cylinder \( (h) = 60 \, \text{cm} \)

\[ \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 10.5 \times 10.5 \times 60 \, \text{cm}^3 \]

\[ = 22 \times 1.5 \times 10.5 \times 60 = 20790 \, \text{cm}^3 \]

Weight of 1 cubic cm (i.e. 1 cm\(^3\)) = 5 g
Total weight of the cylinder = 20790×5 g = 103950 g = \frac{103950}{1000} \text{ kg} = 103.95 \text{ kg}

Q.37. A cylindrical tank has a capacity of 6160 m$^3$. Find its depth if its radius is 14 m. Also find the cost of painting its curved surface at Rs 3 per m$^2$.

Ans. Volume of cylinder = 6160 m$^3$ and radius of cylinder $(r) = 14$ m

Let depth of cylinder be $h$ m.

\[ \frac{22}{7} \times 14 \times 14 \times h = 6160 \Rightarrow h = \frac{6160 \times 7}{22 \times 14 \times 14} = 10 \text{ m} \]

Curved surface area of cylinder = \(2\pi rh = 2 \times \frac{22}{7} \times 14 \times 10 \text{ m}^2 = 880 \text{ m}^2\)

Rate of painting its curved surface = Rs 3 per m$^2$

\[ \therefore \text{ Total cost} = 880 \times 3 = \text{Rs 2640} \]

Q.38. The curved surface area of a cylinder is 4400 cm$^2$ and the circumference of its base is 110 cm. Find the height and the volume of the cylinder.

Ans. Curved surface area = 4400 cm$^2$ and circumference of its base = 110 cm

Let radius be $r$ and height be $h$

Circumference of base = $2\pi r = 110$

\[ \Rightarrow 2 \times \frac{22}{7} r = 110 \Rightarrow r = \frac{110 \times 7}{22 \times 2} = \frac{35}{2} \text{ cm} \]

Curved surface area of cylinder = \(2\pi rh = 4400\)

\[ \Rightarrow 2 \times \frac{22}{7} \times \frac{35}{2} \times h = 4400 \Rightarrow h = \frac{4400 \times 7 \times 2}{2 \times 22 \times 35} = 40 \text{ cm} \]

Volume of the cylinder = \(\pi r^2 h = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 \text{ cm}^3 = 38500 \text{ cm}^3\)

Q.39. The total surface area of a solid cylinder is 462 cm$^2$ and its curved surface area is one-third of its total surface area. Find the volume of the cylinder.

Ans. Total surface area of cylinder = 462 cm$^2$

and curved surface area = \(1/3 \) of total surface area = \(1/3 \times 462 = 154 \text{ cm}^2\)

Let $r$ be the radius and $h$ be its height, then

\[ 2\pi rh = 154 \Rightarrow 2 \times \frac{22}{7} rh = 154 \Rightarrow rh = 154 \times \frac{7}{44} = \frac{49}{2} \]

...(i)

and Area of two bases \(2\pi r^2 = 462 - 154 = 308\)

\[ \Rightarrow 2 \times \frac{22}{7} \times r^2 = 308 \Rightarrow r^2 = \frac{308 \times 7}{2 \times 22} = 49 = (7)^2 \therefore r = 7 \text{ cm} \]
Now putting the value of \( r \) in (i), we get
\[
7 \times h = \frac{49}{2} \Rightarrow h = \frac{49}{2 \times 7} = \frac{7}{2} \text{ cm}
\]

Hence, volume of cylinder \( = \pi r^2 h = \frac{22}{7} \times 7 \times \frac{7}{2} \text{ cm}^3 = 539 \text{ cm}^3 \)

**Q.40.** The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 m², find its volume.

**Ans.** Let \( r \) be the radius and \( h \) be the height of the cylinder. Then

the total surface area of cylinder \( = 1628 \text{ m}^2 \)
\[
\Rightarrow 2\pi r (r + h) = 1628 \Rightarrow 2 \times \frac{22}{7} r \times 37 = 1628
\]
\[
\Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7. \text{ But } r + h = 37 \Rightarrow 7 + h = 37
\]
\[
\therefore \quad h = 37 - 7 = 30 \text{ m}
\]
Volume of the cylinder \( = \pi r^2 h = \frac{22}{7} \times 7 \times 30 \text{ m}^3 = 4620 \text{ m}^3 \)

**Q.41.** A road roller is cylindrical in shape. Its circular end has a diameter 0.7 m and its width is 2 m. Find the least number of complete revolutions that the roller must make in order to level a play-ground measuring 80 m by 44 m.

**Ans.** Diameter of cylinder = 0.7 m ⇒ Radius of cylinder, \( r = \frac{0.7}{2} = 0.35 \text{ m} \)

Width of roller, \( h = 2 \text{ m} \)

Curved surface area of roller \( = 2\pi rh = 2 \times \frac{22}{7} \times 0.35 \times 2 = 4.40 \text{ m}^2 \)
\[
\therefore \quad \text{In 1 revolution, area covered by roller} = 4.40 \text{ m}^2
\]

Area of play-ground \( = 80 \times 44 = 3520 \text{ m}^2 \)
\[
\therefore \quad \text{Number of revolutions} = \frac{3520}{4.40} = 800.
\]

**Q.42.** How many cubic metres of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq. metre.

**Ans.** Depth of well, \( h = 28 \text{ m} \) and diameter of well = 2.8 m
\[
\therefore \quad \text{Radius of well}, \quad r = \frac{2.8}{2} = 1.4 \text{ m}
\]
\[
\therefore \quad \text{Volume of earth dug out} = \pi r^2 h = \frac{22}{7} \times 1.4 \times 1.4 \times 28 = 172.48 \text{ m}^3
\]
Curved surface of well \( = 2\pi rh = 2 \times \frac{22}{7} \times 1.4 \times 28 = 246.4 \, \text{m}^2 \)

Cost of plating at the rate of Rs 4.50 per sq. metre
\[ = 246.4 \times 4.50 = \text{Rs} \, 1108.800 \]

**Q.43.** Find the length of 11 kg copper wire of diameter 0.4 cm. Given one cubic cm of copper weighs 8.4 gram.

**Ans.** Let length of copper wire be \( h \) cm.

Diameter of copper wire \( = 0.4 \, \text{cm} \) then radius of copper wire \( (r) = 0.2 \, \text{cm} \)

\[ \therefore \text{Volume of copper wire (cylinder)} = \pi r^2 h = \frac{22}{7} \times 0.2 \times 0.2 \times h \]
\[ = \frac{22}{7} \times \frac{2}{10} \times \frac{2}{10} \times h = \frac{88}{700} \, h \, \text{cm}^3 \]

Weigh of 1 cm\(^3\) of wire = 8.4 grams

\[ \therefore \text{Total weight of wire} = \frac{88}{700} \times h \times 8.4 = \frac{88}{700} \times h \times \frac{84}{10} \, \text{grams} \]

But, weight of wire = 11 kg = 11\times1000 = 11000 \, \text{gm}

\[ \therefore \frac{88}{700} \times h \times \frac{84}{10} = 11000 \]
\[ h = \frac{11000 \times 700 \times 10}{88 \times 84} \Rightarrow h = \frac{125 \times 100 \times 10}{12} = \frac{125 \times 25 \times 10}{3} \]
\[ \Rightarrow h = \frac{31250}{3} \, \text{cm} = \frac{31250}{3 \times 100} \, \text{m} = \frac{625}{6} \, \text{m} \therefore h = 104.17 \, \text{m} \]

**Q.44.** A cylinder has a diameter of 20 cm. The area of curved surface is 100 \, \text{cm}^2 \, (\text{sq. cm}). Find :

(i) the height of the cylinder correct to one decimal place.

(ii) the volume of the cylinder correct to one decimal place.

**Ans.** (i) Diameter of cylinder = 20 cm then radius \( r = \frac{20}{2} = 10 \, \text{cm} \)

Let height of cylinder be \( h \) cm.

Curved surface area of cylinder = 100 \, \text{cm}^2
\[ 2\pi rh = 100 \]
\[ 2 \times \frac{22}{7} \times 10 \times h = 100 \]
\[ h = \frac{100 \times 7}{2 \times 22 \times 10} = \frac{35}{22} \, \text{cm} = 1.6 \]
(ii) If \( h \) is taken as 1.6 cm

Then volume of cylinder \( = \pi r^2 h = \frac{22}{7} \times 10 \times 10 \times 1.6 = 502.9 \) cm\(^3\)

Hence, (i) height of cylinder = 1.6 cm (ii) volume of cylinder 502.9 cm\(^3\).

**Q.45.** A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 2 mm thick all round. Find the weight in kilogram, of 2 metres of the pipe, if 1 cm\(^3\) of the metal weighs 7.7 g.

**Ans.** Inner diameter = 5 cm ∴ Inner radius \( \eta = \frac{5}{2} = 2.5 \) cm

Thickness = 2 mm = 0.2 cm

∴ Outer radius, \( r_2 = 2.5 \) cm + 0.2 cm = 2.7 cm

Length of pipe = 2 metres = 200 cm

∴ Volume of metal in the pipe \( = \pi r_2^2 h - \pi \eta^2 h \)

\[ = \pi h [r_2^2 - \eta^2] = \frac{22}{7} \times 200 [(2.7)^2 - (2.5)^2] = \frac{22}{7} \times 200 [7.29 - 6.25] \]

\[ = \frac{22}{7} \times 200 \times 1.04 = \frac{22}{7} \times 200 \times \frac{104}{100} = \frac{44 \times 104}{7} \] cm\(^3\)

1 cm\(^3\) of metal weighs = 7.7 gm

∴ \[ \frac{44 \times 104}{7} \] of metal weighs \[ = \frac{44 \times 104}{7} \times 7.7 = \frac{44 \times 104 \times 1.1}{100} = 5033.6 \] gm

\[ = \frac{5033.6}{1000} \] kg = 5.0336 kg = 5.034 kg

**Q.46.** Find the total surface area of a hollow cylinder open at both ends, if its length is 12 cm, external diameter is 8 cm and the thickness is 2 cm.

**Ans.** Length of hollow cylinder \( (h) = 12 \) cm

External diameter of cylinder = 8 cm

∴ External radius of cylinder \( (r) = \frac{8}{2} \) cm = 4 cm

Thickness of the cylinder = 2 cm

Inner radius of cylinder = \( 4 - 2 = 2 \) cm

Thus, total surface area of cylinder

\[ = [2 \pi Rh + 2 \pi rh + 2 (\pi R^2 - \pi r^2)] \] cm\(^2\)

\[ = \left[ \left( 2 \times \frac{22}{7} \times 4 \times 12 + 2 \times \frac{22}{7} \times 2 \times 12 \right) + 2 \left( \frac{22}{7} \times 4 \times 4 - \frac{22}{7} \times 2 \times 2 \right) \right] \] cm\(^2\)
= \left[ \frac{2112}{7} + \frac{1056}{7} \right] + 2 \left( \frac{352}{7} - \frac{88}{7} \right) \text{ cm}^2

= \left[ \frac{3168}{7} + 2 \times \frac{264}{7} \right] \text{ cm}^2 = \frac{3168}{7} + \frac{528}{7} = \frac{3696}{7} = 528 \text{ cm}^2

Q.47. Water is flowing at the rate of 8 m per second through a circular pipe whose internal diameter is 2 cm, into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the water level in 30 minutes.

Ans. Diameter of pipe = 2 cm, ∴ Radius of pipe = 1 cm
Rate of water flow = 8 m/sec
Time taken = 30 minutes
∴ Length of water flow into the pipe \((h) = 30 \times 60 \times 8 = 14400 \text{ m}\)

∴ Volume of water \(= \pi r^2h = \frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 14400 \text{ m}^3 = \frac{144 \times 22}{700} \text{ m}^3\)

Radius of the base of the cylindrical tank = 40 cm = \(\frac{40}{100} \text{ m} = \frac{2}{5} \text{ m}\)

Let \(h\) be the water increased in the tank
∴ \(\pi r^2h = \frac{144 \times 22}{700} \Rightarrow \frac{22}{7} \times \frac{2}{5} \times h = \frac{144 \times 22}{700}\)
∴ \(h = \frac{144 \times 22 \times 5 \times 5}{700 \times 22 \times 2 \times 2} = 9 \text{ m}\)
Hence, height of water is 9 m.

Q.48. The sum of the inner and the outer curved surfaces of a hollow metallic cylinder is 1056 cm², and the volume of material in it is 1056 cm³. Find the internal and external radii. Given that the height of the cylinder is 21 cm.

Ans. Let \(r\) and \(R\) be the inner and outer radii and \(h\) be their height
∴ \(2\pi Rh + 2\pi rh = 1056\)
\(\Rightarrow 2\pi h (R + r) = 1056\)
\(\Rightarrow 2 \times \frac{22}{7} \times 21 (R + r) = 1056\)
\(\Rightarrow 132 (R + r) = 1056 \Rightarrow R + r = \frac{1056}{132} = 8 \quad \ldots(i)\)

Again \(\pi R^2h - \pi r^2h = 1056 \Rightarrow \pi h (R^2 - r^2) = 1056\)
\(\Rightarrow \frac{22}{7} \times 21 (R^2 - r^2) = 1056\)
\(\Rightarrow 66 (R^2 - r^2) = 1056\)
\[ \Rightarrow R^2 - r^2 = \frac{1056}{66} = 16 \]
\[ \Rightarrow (R + r) (R - r) = 16 \]
\[ \Rightarrow 8 (R - r) = 16 \]
\[ \Rightarrow R - r = \frac{16}{8} = 2 \]
\[ R + r = 8 \quad \text{...(ii)} \]
\[ R - r = 2 \quad \text{...(iii)} \]
Adding (ii) and (iii), we get
\[ 2R = 10 \Rightarrow R = 5 \]
Subtracting eqn. (iii) from (ii), we get
\[ 2r = 6 \Rightarrow r = 3 \]
Hence, outer radius of cylinder = 5 cm and inner radius of cylinder = 3 cm.