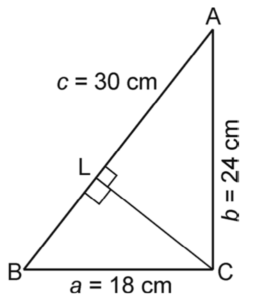
9 PERIMETER AND AREA OF PLANE FIGURES

Q.1. Find the area of a triangle whose sides are 18 cm, 24 cm and 30 cm. Also, find the length of altitude corresponding to the largest side of the triangle.

Ans. Let ABC be the triangle



Let $a = 18$ cm, $b = 24$ cm, $c = 30$ cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{18+24+30}{2} \text{ cm} = \frac{72}{2} \text{ cm}$$

$$\therefore \quad s = 36 \text{ cm}$$

Using Hero's Formula

$$\text{Area of } \Delta\text{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-18)(36-24)(36-30)} \text{ cm}^2$$

$$= \sqrt{36(18)(12)(6)} \text{ cm}^2$$

$$= \sqrt{2\times2\times3\times3\times2\times3\times3\times2\times2\times3\times2\times3} \text{ cm}^2$$

$$= 2\times2\times2\times3\times3\times3 \text{ cm}^2 = 8\times27 \text{ cm}^2 = 216 \text{ cm}^2$$

(ii) Let CL be altitude on the largest side AB of ΔABC

Area of $\Delta\text{ABC} = 216 \text{ cm}^2$ [From (i)]

$$\Rightarrow \frac{1}{2}\times\text{base}\times\text{altitude} = 216 \text{ cm}^2 \Rightarrow \frac{1}{2}\times\text{AB}\times\text{CL} = 216 \text{ cm}^2$$

$$\Rightarrow \frac{1}{2}\times30\times\text{CL} = 216 \text{ cm}^2 \Rightarrow 15\times\text{CL} = 216 \text{ cm}^2$$

$$\Rightarrow \text{CL} = \frac{216}{15} \text{ cm} = 14.4 \text{ cm}$$

Hence, altitude on the largest side is 14.4 cm.

Q.2. Find the area of the triangle whose sides are 13 cm, 14 cm and 15 cm. Also find the height of the triangle corresponding to the longest side.

Ans. Let $a = 13$ cm, $b = 14$ cm, and $c = 15$ cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21 \text{ cm}$$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)} = \sqrt{21 \times 8 \times 7 \times 6}\ \text{cm}^2$

$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} = 2 \times 2 \times 3 \times 7 = 84\ \text{cm}^2$

Let the height on the longest side be h then,

Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height} \Rightarrow 84 = \frac{1}{2} \times 15 \times h$

$\therefore$ Height of triangle, $h = \frac{84 \times 2}{15} = \frac{56}{5} = 11.2$ cm

Q.3. Find the area of the triangle whose sides are 30 cm, 24 cm and 18 cm. Also find the length of the altitude corresponding to the smallest side of the triangle.

Ans. Let sides of triangle are $a = 30$ cm, $b = 24$ cm and $c = 18$ cm, then

$s = \frac{a+b+c}{2} = \frac{30+24+18}{2} = \frac{72}{2} = 36$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{36(36-30)(36-24)(36-18)}$

$= \sqrt{36 \times 6 \times 12 \times 18}\ \text{cm}^2 = \sqrt{2 \times 2 \times 3 \times 3 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 3 \times 3}\ \text{cm}^2$

$= 2 \times 3 \times 2 \times 3 \times 2 \times 3 = 216\ \text{cm}^2$

Let the length of altitude on the smallest side of 18 cm $= h.$

Then area of triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$\Rightarrow 216 = \frac{1}{2} \times 18 \times h \Rightarrow 9h = 216 \Rightarrow h = \frac{216}{9} = 24$ cm

$\therefore$ Altitude of triangle $= 24$ cm

Q.4. The lengths of the sides of triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle.

Ans. Ratio of sides $= 3:4:5$ and perimeter of triangle $= 144$ cm

Let the sides are $3x$, $4x$ and $5x$

$\therefore$ $3x + 4x + 5x = 144 \Rightarrow 12x = 144 \Rightarrow x = \frac{144}{12} = 12$

$\therefore$ Sides are $3 \times 12, 4 \times 12, 5 \times 12$ i.e. 36 cm, 48 cm and 60 cm.

$s = \frac{\text{sum of sides}}{3} = \frac{144}{2} = 72$

$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72(72-36)(72-48)(72-60)}$

$= \sqrt{72 \times 36 \times 24 \times 12}\ \text{cm}^2$

$= \sqrt{36 \times 2 \times 36 \times 2 \times 12 \times 12}\ \text{cm}^2 = 2 \times 36 \times 12\ \text{cm}^2 = 864\ \text{cm}^2$

Q.5. The base of a triangular field is twice its altitude. If the cost of cultivating the field at Rs 14.50 per 100 m^2 is Rs 52,200, find its base and altitude.

Ans. Let altitude of the triangle $= x$ and base of triangle $= 2x$

$\therefore$ Area of triangle $= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times 2x \times x = x^2$ ...(i)

Total cost of cultivating of the field $=$ Rs $52,200$

Rate of cultivating $=$ Rs 14.50 per $100\ \text{m}^2$

$\therefore$ Area of the field $= \frac{52200 \times 100}{14.50} = \frac{52200 \times 100 \times 100}{1450}\ \text{m}^2$

$= 360000\ \text{m}^2$ ...(ii)

From (i) and (ii), we get

$\therefore$ $x^2 = 360000 \Rightarrow x^2 = (600)^2 \Rightarrow x = 600$

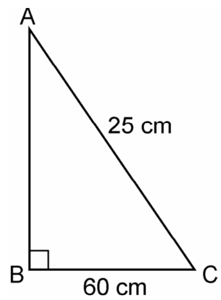
Hence, base $= 2 \times 600 = 1200$ m and altitude $= 600 \times 1 = 600$ m

Q.6. The perimeter of a right triangle is 60 cm and its hypotenuse is 25 cm. Find the area of the triangle.

Ans. In ΔABC, $\angle B = 90°$ hypotenuse AC = 25 cm and perimeter of triangle = 60 cm.

$\therefore$ $AB + BC = 60 - 25 = 35$ cm

Let base BC $= x$ cm then altitude $AB = 35 - x$



But $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow$ $(25)^2 = (35 - x)^2 + x^2 \Rightarrow 625 = 1225 - 70x + x^2 + x^2$

$\Rightarrow$ $2x^2 - 70x + 1225 - 625 = 0 \Rightarrow 2x^2 - 70x + 600 = 0$

$\Rightarrow$ $x^2 - 35x + 300 = 0$ (Dividing both sides by 2)

$\Rightarrow$ $x^2 - 20x - 15x + 300 = 0 \Rightarrow x(x - 20) - 15(x - 20) = 0$

$\Rightarrow$ $(x - 20)(x - 15) = 0$

If $x - 20 = 0$, then $x = 20$. If $x - 15 = 0$, then $x = 15$

If $x = 20$, then base $= 20$ cm and altitude $= 35 - 20 = 15$ cm

If $x = 15$, then base $= 15$ cm and altitude $= 35 - 15 = 20$ cm

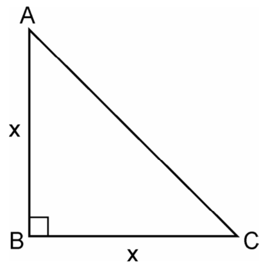
Hence, sides are 15 cm and 20 cm.

Q.7. Find the length of hypotenuse of an isosceles right angled triangle having an area of 200 cm^2 (Take $\sqrt{2} = 1.414$).

Ans. In right angled isosceles triangle ABC, $\Rightarrow B = 90°$ and AB = BC

Let $AB = BC = x$ cm

$\therefore$ Area $= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times x \times x = \frac{x^2}{2}$



But area of $\Delta ABC = 200\ cm^2$ (Given)

$\therefore$ $\frac{x^2}{2} = 200 \Rightarrow x^2 = 400 = (20)^2$

$\therefore$ $x = 20$

Hypotenuse AC $= \sqrt{AB^2 + BC^2} = \sqrt{(20)^2 + (20)^2}$

$= \sqrt{400 + 400} = \sqrt{800} = \sqrt{2 \times 400}$

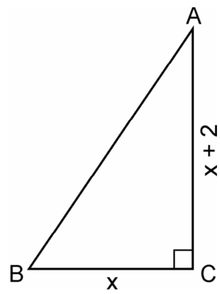
$= 20 \times \sqrt{2}$ cm $= 20 \times 1.414$ cm

$= 28.280$ cm $= 28.28$ cm

Q.8. The lengths of two sides of a right triangle containing the right angle differ by 2 cm. If the area of the triangle is 24 cm^2, find the perimeter of the triangle.

Ans. In $\Delta ABC, \angle C = 90°$

Let $BC = x$ cm then $AC = x + 2$ cm



$\therefore$ Area of triangle ABC $= \frac{1}{2} \times BC \times AC$

$\Rightarrow$ $24 = \frac{1}{2} x(x+2) \Rightarrow 48 = x^2 + 2x$

$\Rightarrow$ $x^2 + 2x - 48 = 0 \Rightarrow x^2 + 8x - 6x - 48 = 0$

$\Rightarrow$ $x(x+8) - 6(x+8) = 0 \Rightarrow (x+8)(x-6) = 0$

If $x + 8 = 0$, then $x = -8$ which is not possible.

If $x - 6 = 0$, then $x = 6$

$\therefore$ $BC = 6$ cm and $AC = 6 + 2 = 8$ cm

But $AB^2 = BC^2 + AC^2$ (Using Pythagoras Theorem)

$= (6)^2 + (8)^2 = 36 + 64 = 100 = (10)^2$

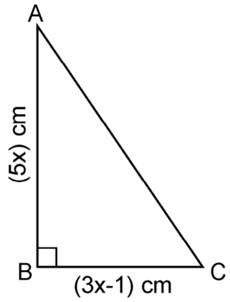
$\therefore$ $AB = 10$ cm

Perimeter of the triangle ABC $= AB + BC + CA = 10 + 6 + 8$ cm $= 24$ cm

Q.9. The sides of a right angled triangle containing the right angle are $(5x)$ cm and $(3x-1)$ cm. If its area is 60 cm^2, find its perimeter.

Ans. In ΔABC, $\angle B = 90°$, Area of triangle $= 60 \text{ cm}^2$ (Given)

$AB = (5x)$ cm and $BC = (3x-1)$ cm



$\therefore$ Area of triangle ABC $= \frac{1}{2} BC \times AB$

$\Rightarrow 60 = \frac{1}{2} \times (3x-1) \times 5x \Rightarrow 120 = 15x^2 - 5x$

$\Rightarrow 15x^2 - 5x - 120 = 0 \Rightarrow 3x^2 - x - 24 = 0$

$\Rightarrow 3x^2 - 9x + 8x - 24 = 0$

$\Rightarrow 3x(x-3) + 8(x-3) = 0 \Rightarrow (x-3)(3x+8) = 0$

If $x - 3 = 0$, then $x = 3$

or $3x + 8 = 0$, then $x = -\frac{8}{3}$, which is not possible.

$\therefore AB = 5x = 5 \times 3 = 15$ cm and $BC = 3x - 1 = 3 \times 3 - 1 = 9 - 1 = 8$ cm

But $AC^2 = AB^2 + BC^2 = 15^2 + 8^2$ (Using Pythagoras Theorem)

$= 225 + 64 = 289$

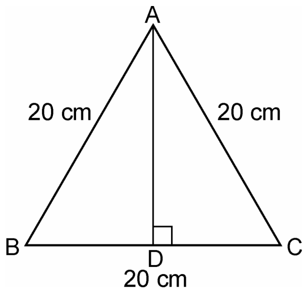
$\Rightarrow AC = \sqrt{289} = 17$

Now, perimeter $= (15 + 8 + 17)$ m $= 40$ m

Q.10. Calculate the area and the height of an equilateral triangle whose perimeter is 60 cm.

Ans. Perimeter of $\Delta ABC = 60$ cm (Given)

Thus each side of triangle $= \frac{60}{3} = 20$ cm



Altitude AD $= \frac{\sqrt{3}}{2} a = \frac{1.732}{2} \times 20 = 17.32$ cm

Area of $\Delta ABC = \frac{\sqrt{3}}{4} a^2 = \frac{1.732}{4} (20^2)$ cm^2

$= \frac{1.732}{4} \times 400 = 173.2 \text{ cm}^2$

Q.11. The area of an equilateral triangle is $36\sqrt{3}$ sq. m. Find its perimeter.

Ans. Area of equilateral triangle $= 36\sqrt{3}$ sq.cm. (Given)

$$\Rightarrow \quad \frac{\sqrt{3}}{4}\times(\text{side})^2 = 36\sqrt{3} \Rightarrow \sqrt{3}\times(\text{side})^2 = 36\sqrt{3}\times 4$$

$$\Rightarrow \quad (\text{side})^2 = \frac{36\sqrt{3}\times 4}{\sqrt{3}} = 36\times 4$$

Taking square root of both sides, we get

$$\sqrt{(\text{side})^2} = \sqrt{36\times 4}$$

$$\sqrt{(\text{side})^2} = \sqrt{(3\times 3)\times(4\times 4)}$$

$\therefore \quad \text{side} = 3\times 4 = 12$ cm

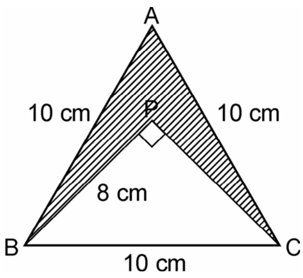
Perimeter of equilateral $\Delta ABC = 3\times\text{side} = 3\times 12 \text{ cm} = 36$ cm.

Q.12. In the given figure, ΔABC is an equilateral triangle having each side equal to 10 cm and ΔPBC is right angled at P in which PB = 8 cm. Find the area of the shaded region.

Ans. ΔABC is an equilateral triangle whose each side $= 10$ cm

ΔBPC is right angled triangle in which $\angle P = 90°$ and $PB = 8$ cm

In ΔPBC, $BC^2 = PB^2 + PC^2$ (Using Pythagoras Theorem)



$$\Rightarrow \quad (10)^2 = (8)^2 + PC^2 \Rightarrow 100 = 64 + PC^2$$

$$\Rightarrow \quad PC^2 = 100 - 64 = 36 = (6)^2$$

$\therefore \quad PC = 6$ cm

$$\text{Area of } \Delta PBC = \frac{1}{2}\times PB\times PC = \frac{1}{2}\times 8\times 6 = 24 \text{ cm}^2$$

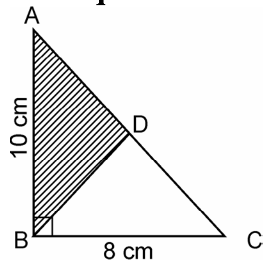
$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}a^2 = \frac{1.732}{4}\times(10)^2 \text{ cm}^2$$

$$= \frac{1.732\times 100}{4} = 0.433\times 100 \text{ cm}^2 = 43.3 \text{ cm}^2$$

Hence, area of shaded portion $=$ Area of $\Delta ABC -$ Area of ΔPBC

$$= 43.3 - 24.0 = 19.3 \text{ cm}^2$$

Q.13. The given figure shows a right angled triangle ABC and an equilateral triangle BCD. Find the area of the shaded portion.



Ans. In right angled ΔABC, $\angle B = 90°$

$AB = 10$ cm, $BC = 8$ cm

$\therefore$ Area of $\Delta ABC = \frac{1}{2} AB \times BC = \frac{1}{2} \times 10 \times 8 = 40 \text{ cm}^2$

Side of equilateral $\Delta BCD = 8$ cm

$\therefore$ Area of equilateral triangle BCD $= \frac{\sqrt{3}}{4} a^2 = \frac{1.732}{4} \times (8)^2 \text{ cm}^2$

$= 0.433 \times 64 = 27.712 \text{ cm}^2$

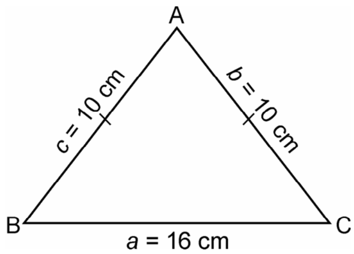
Hence, area of shaded portion $= (40.000 - 27.712) \text{ cm}^2 = 12.288 \text{ cm}^2 = 12.29 \text{ cm}^2$.

Q.14. Find the area of an isosceles triangle whose perimeter is 36 cm and base is 16 cm.

Ans. ABC is isosceles triangle in which AB = AC.

Let, AB = AC = x cm, base BC = 16 cm

Perimeter of $\Delta ABC = 36$ cm (Given)



$AB + AC + BC = 36$ cm

$\Rightarrow x + x + 16 = 36 \Rightarrow 2x + 16 = 36$

$\Rightarrow 2x = 36 - 16 \Rightarrow 2x = 20$

$\Rightarrow x = \frac{20}{2} = 10$

$\therefore AB = x \text{ cm} = 10 \text{ cm}$ and $AC = x \text{ cm} = 10 \text{ cm}$

Let $a = 16$ cm, $b = 10$ cm and $c = 10$ cm

$s = \frac{a+b+c}{2} = \frac{16+10+10}{2} \text{ cm} = \frac{36}{2} = 18 \text{ cm}$

Using Hero's Formula

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-16)(18-10)(18-10)} \text{ cm}^2$

$= \sqrt{18(2)(8)(8)} \text{ cm}^2$

$= \sqrt{(2\times3\times3)\times(2)\times(2\times2\times2)\times(2\times2\times2)} \text{ cm}^2$

$= \sqrt{2\times3\times3\times2\times2\times2\times2\times2\times2\times2} \text{ cm}^2$

$= \sqrt{(2\times2)\times(2\times2)\times(2\times2)\times(2\times2)\times(3\times3)} \text{ cm}^2$

$= 2\times2\times2\times2\times3 \text{ cm}^2 = 48 \text{ cm}^2$

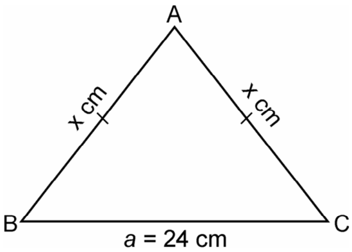
Q.15. The base of an isosceles triangle is 24 cm and its area is 192 sq. cm. Find its perimeter.

Ans. Let ABC be an isosceles triangle in which

AB = AC

Let AB = AC = x cm, base BC = 24 cm

Let $a = 24$ cm, $b = x$ cm and $c = x$ cm



$\therefore \quad s = \frac{a+b+c}{2} = \frac{24+x+x}{2} \text{ cm} = \frac{2(12+x)}{2} \text{ cm}$

$= (12+x)$ cm

Area of ΔABC $= 192 \text{ cm}^2$ (Given)

$\sqrt{s(s-a)(s-b)(s-c)} = 192 \text{ cm}^2$

$\Rightarrow \sqrt{(12+x)(12+x-24)(12+x-x)(12+x-x)} = 192$

$\Rightarrow \sqrt{(12+x)(x-12)(12)(12)} = 192 \Rightarrow 12\sqrt{(12+x)(x-12)} = 192$

$\Rightarrow \sqrt{(12+x)(x-12)} = \frac{192}{12} = 16$

Squaring both sides, we get

$\Rightarrow (\sqrt{12+x)(x-12)})^2 = (16)^2 \Rightarrow (12+x)(x-12) = 16\times16$

$\Rightarrow 12x - 144 + x^2 - 12x = 256 \Rightarrow x^2 - 144 = 256$

$\Rightarrow x^2 = 256 + 144 = 400$

Squaring root both sides, we get

$\Rightarrow \sqrt{x^2} = \sqrt{400} = \sqrt{20\times20}$

$\Rightarrow x = 20 \;\therefore\; a = 24$ cm

$b = x$ cm $\Rightarrow b = 20$ cm and $c = x$ cm $\Rightarrow c = 20$ cm

$\therefore$ Perimeter of ΔABC $= a+b+c = (24+20+20)$ cm $= 64$ cm

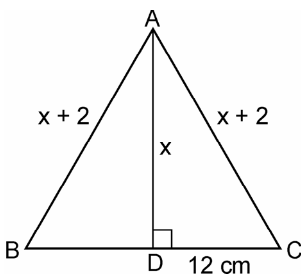
Q.16. Each of the equal sides of an isosceles triangle is 2 cm more than its height and the base of the triangle is 12 cm. Find the area of the triangle.

Ans. Let height AD $= x$ cm then AB = AC $= (x+2)$ cm

Base BC = 12 cm $\because$ AD $\perp$ BC

$\therefore \quad \text{BD} = \text{DC} = \frac{12}{2} = 6$ cm

$\therefore$ In ΔABD, $\text{AB}^2 = \text{BD}^2 + \text{AD}^2$



$\Rightarrow \quad (x+2)^2 = (6)^2 + x^2$

$\Rightarrow \quad x^2 + 4x + 4 = 36 + x^2 \Rightarrow x^2 + 4x - x^2 = 36 - 4$

$\Rightarrow \quad 4x = 32 \Rightarrow x = \frac{32}{4} = 8 \;\therefore\; \text{AD} = 8 \text{ cm}$

Area of $\Delta\text{ABC} = \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times \text{BD} \times \text{AD} = \frac{1}{2} \times 12 \times 8 \text{ cm}^2 = 48 \text{ cm}^2$

Q.17. Find the area of an isosceles triangle, each of whose equal sides is 13 cm and base 24 cm.

Ans. ΔABC is an isosceles triangle in which $\text{AB} = \text{AC} = 13$ cm and $\text{BC} = 24$ cm.
$\text{AD} \perp \text{BC}$ which bisects BC at D. Then $\text{BD} = \text{DC}$

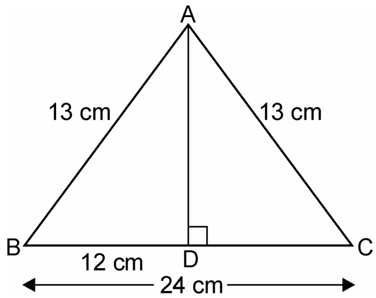
$= \frac{1}{2}\text{BC} = \frac{1}{2} \times 24 = 12 \text{ cm}$

In right ΔABD, $\text{AB}^2 = \text{BD}^2 + \text{AD}^2$ (Using Pythagoras theorem)

$\Rightarrow \quad (13)^2 = (12)^2 + \text{AD}^2 \Rightarrow 169 = 144 + \text{AD}^2$

$\Rightarrow \quad \text{AD}^2 = 169 - 144 = 25 = (5)^2 \;\therefore\; \text{AD} = 5 \text{ cm}$

Area of $\Delta\text{ABC} = \frac{1}{2}\text{BC} \times \text{AD} = \frac{1}{2} \times 24 \times 5 = 60 \text{ cm}^2$



Q.18. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.

Ans. ΔABC is an isosceles triangle in which $\text{AB} = \text{AC}$ and $\text{AD} \perp \text{BC}$
Perimeter of triangle $= 32$ cm
$\text{AD} = 8$ cm

Let $\text{AB} = \text{AC} = a$ and $\text{BC} = b$, then $\text{BD} = \text{DC} = \frac{b}{2}$

In right ΔABD, $\text{AB}^2 = \text{BD}^2 + \text{AD}^2$ (Using Pythagoras Theorem)

$\Rightarrow \quad a^2 = \frac{b^2}{4} + (8)^2 \Rightarrow 4a^2 = b^2 + 256$

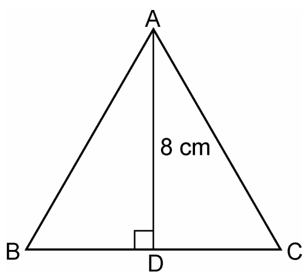
$\Rightarrow \quad 4a^2 - b^2 = 256$

$\Rightarrow \quad (2a+b)(2a-b) = 256 \qquad \text{...(i)}$

But, $\text{AB} + \text{AC} + \text{BC} = 32$

$\Rightarrow \quad a + a + b = 32$

$\Rightarrow \quad 2a + b = 32 \qquad \text{...(ii)}$



Dividing (i) by (ii), we get

$$2a-b=\frac{256}{32}=8 \qquad \ldots\text{(iii)}$$

Adding (ii) and (iii), we get

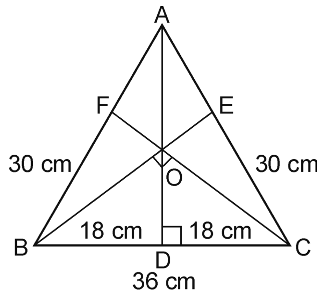
$$4a=40 \Rightarrow a=\frac{40}{4}=10$$

Subtracting eqn. (iii) from (ii),

$$2b=24 \Rightarrow b=12$$

$$\text{Area of } \Delta ABC=\frac{BC\times AD}{2}=\frac{1}{2}\times 12\times 8=48\text{ cm}^2$$

Q.19. AD is altitude of an isosceles triangle ABC in which AB = AC = 30 cm and BC = 36 cm. A point O is marked on AD in such a way that $\angle BOC=90°$, Find the area of quadrilateral ABOC.



Ans. In an isosceles ΔABC, AB = AC = 30 cm, BC = 36 cm

AD is altitude on BC

O is an point on AD such that $\angle BOC=90°$

$\because$ $AD\perp BC$ and $AB=AC$

$\therefore$ D is mid-point of BC also $BD=DC=\frac{36}{2}=18$ cm

In ΔADB,

$$AB^2=BD^2+AD^2 \qquad \text{(By Pythgoras theorem)}$$

$$\text{Length } AD=\sqrt{AB^2-BD^2}=\sqrt{30^2-18^2}=\sqrt{900-324}=\sqrt{576}=24\text{ cm}$$

$$\therefore \text{Area of } \Delta ABC=\frac{1}{2}BC\times AD=\frac{1}{2}\times 36\times 24\text{ cm}^2=432\text{ cm}^2$$

In ΔOBC, $OB=OC$

Let $OB=OC=a$, $BC=36$ cm

then $BC^2=a^2+a^2=2a^2$

Using Pythagoras Theorem

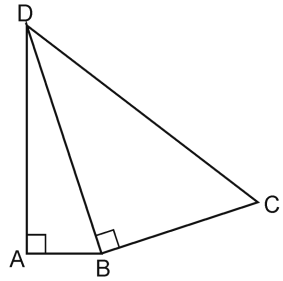
$$(36)^2=2a^2 \Rightarrow a^2=\frac{(36)^2}{2}$$

$$\text{Area of } \Delta OBC=\frac{1}{2}a\times a=\frac{1}{2}a^2=\frac{1}{2}\times\frac{(36)^2}{2}=\frac{1296}{4}=324\text{ cm}^2$$

Hence, area of quad ABOC $=$ Area of $\Delta ABC-$ Area of ΔOBC

$$=(432-324)\text{ cm}^2=108\text{ cm}^2$$

Q.20. Find the area and perimeter of quadrilateral ABCD, given below; if, AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and $\angle DBC = 90°$.



Ans. In ΔDBC, right angled at B

$DB = 12$ cm, $DC = 13$ cm

$DB^2 + BC^2 = DC^2$ [Using Pythagoras Theorem]

$(12)^2 + BC^2 = (13)^2 \Rightarrow 144 + BC^2 = 169$

$BC^2 = 169 - 144 = 25$

Taking square root of both sides, we get

$\sqrt{BC^2} = \sqrt{25} \Rightarrow BC = 5$

Area of quadrilateral ABCD

= Area of ΔABD + Area of ΔDBC ...(i)

To find area of ΔADB,

Let $a = 12$ cm, $b = 10$ cm, $c = 8$ cm

$$s = \frac{a+b+c}{2} = \frac{12+10+8}{2} \text{ cm} = \frac{30}{2} \text{ cm} = 15 \text{ cm}$$

Using Hero's Formula,

$$\text{Area of } \Delta ABD = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

$$= \sqrt{15\,(15-12)\,(15-10)\,(15-8)} \text{ cm}^2$$

$$= \sqrt{15\,(3)\,(5)\,(7)} \text{ cm}^2 = \sqrt{5\times3\times3\times5\times7} \text{ cm}^2$$

$$= \sqrt{(5\times5)\times(3\times3)\times7} \text{ cm}^2 = 5\times3\times\sqrt{7} \text{ cm}^2$$

$$= 15\times2.645 = 39.675 \text{ cm}^2$$

$$\text{Area of } \Delta DBC = \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times BC\times DB$$

$$= \frac{1}{2}\times5\times12 \text{ cm}^2 = 5\times6 \text{ cm}^2 = 30 \text{ cm}^2$$

$\therefore$ From (i), we get

$\therefore$ Area of quadrilateral ABCD = Area of ΔABD + Area of ΔDBC

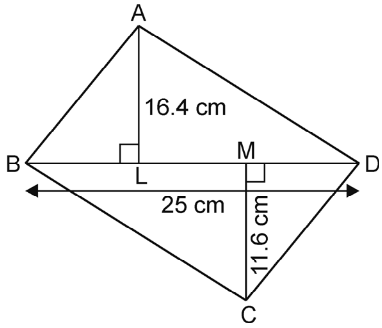
$= 39.675 \text{ cm}^2 + 30 \text{ cm}^2$

$= 69.675 \text{ cm}^2 = 69.7 \text{ cm}^2$

(ii) Perimeter of quadrilateral ABCD $= AB + BC + DC + AD$

$= 8 \text{ cm} + 5 \text{ cm} + 13 \text{ cm} + 10 \text{ cm} = 36 \text{ cm}$

Q.21. Find the area of a quadrilateral one of whose diagonals is 25 cm long and the lengths of perpendiculars from the other two vertices one 16.4 cm and 11.6 cm respectively.



Ans. In quadrilateral ABCD, diagonal BD = 25 cm

$AL \perp BD$ and $CM \perp BD$

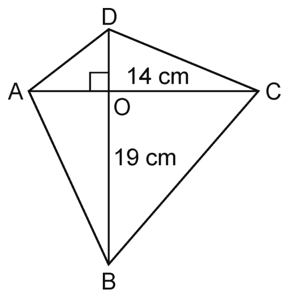
$AL = 16.4$ cm, $CM = 11.6$ cm

$\therefore$ Area of quadrilateral ABCD

$$= \frac{1}{2}(AL + CM) \times BD = \frac{1}{2}(16.4 + 11.6) \times 25 \text{ cm}^2$$

$$= \frac{1}{2} \times 28 \times 25 = 350 \text{ cm}^2$$

Q.22. The diagonals of a quadrilateral intersect each other at right angles. If the lengths of these diagonals be 14 cm and 19 cm respectively, find the area of the quadrilateral.

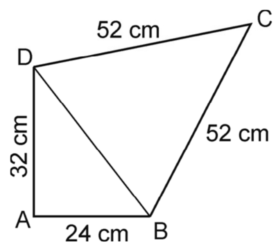


Ans. In quadrilateral ABCD, diagonals AC and BD intersect each other at O at right angles, and AC = 14 cm, BD = 19 cm

Hence, area of quadrilateral ABCD $= \frac{1}{2} AC \times BD$

$$= \frac{1}{2} \times 14 \times 19 \text{ cm}^2 = 133 \text{ cm}^2$$

Q.23. Calculate the area of quadrilateral ABCD in which : AB = 24 cm, AD = 32 cm, $\angle BAD = 90°$ and BC = CD = 52 cm.



Ans. In quadrilateral ABCD, diagonal BD divides it into two triangles ΔABD and ΔBCD.

In $\Delta ABD, \angle BAD = 90°$

$\therefore$ $BD^2 = AD^2 + AB^2$ (Using Pythagoras Theorem)

$$= (32)^2 + (24)^2 = 1024 + 576 = 1600 = (40)^2$$

$\therefore$ $BD = 40$ cm

Area of $\Delta ABD = \frac{1}{2} \times AB \times AD = \frac{1}{2} \times 24 \times 32 \text{ cm}^2 = 384 \text{ cm}^2$

In ΔBCD, sides are $BC = CD = 52$ cm and $BD = 40$ cm

$\therefore$ $s = \frac{a+b+c}{2} = \frac{52+52+40}{2} = \frac{144}{2} = 72$

$\therefore$ Area of $\Delta BCD = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72\,(72-52)\,(72-52)\,(72-40)}$

$= \sqrt{72 \times 20 \times 20 \times 32}$

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2 \times 20^2}\ \text{cm}^2$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 20\ \text{cm}^2 = 960\ \text{cm}^2$

Hence, area of quadrilateral ABCD $= \text{ar. }(\Delta ABD) + \text{ar. }(\Delta BCD)$

$= 384\ \text{cm}^2 + 960\ \text{cm}^2 = 1344\ \text{cm}^2$

Q.24. Calculate the area of quadrilateral ABCD in which ΔBCD is equilateral with each side equal to 26 cm, $\angle BAD = 90°$ and AD = 24 cm.

Ans. In quadrilateral ABCD, diagonal BD divides into two triangles : ΔABD and ΔBCD.

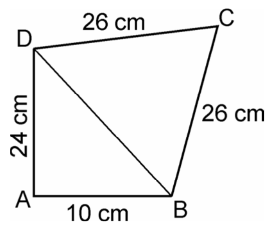
In ΔABD, $\angle BAD = 90°$

$\therefore \quad BD^2 = AD^2 + AB^2$ (Using Pythagoras Theorem)

$\Rightarrow \quad (26)^2 = (24)^2 + AB^2 \Rightarrow 676 = 576 + AB^2$

$\Rightarrow \quad AB^2 = 676 - 576 = 100$

$\Rightarrow \quad AB^2 = (10)^2 \Rightarrow AB = 10\ \text{cm}$



Area of $\Delta ABD = \frac{1}{2} \times AB \times AD = \frac{1}{2} \times 10 \times 24 = 120\ \text{cm}^2$

Area of equilateral ΔBCD whose each side is 26 cm.

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1.732}{4} \times (26)^2\ \text{cm}^2$

$= \frac{1.732}{4} \times 26 \times 26$

$= 0.433 \times 26 \times 26\ \text{cm}^2 = 292.71\ \text{cm}^2$

Hence, area of quadrilateral ABCD $= \text{ar. }(\Delta ABD) + \text{ar. }(\Delta BCD)$

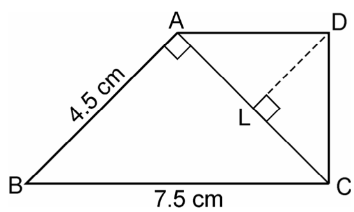
$= 120\ \text{cm}^2 + 292.71\ \text{cm}^2 = 412.71\ \text{cm}^2$

Q.25. In the adjoining figure, ΔABC is right angled at A, BC = 7.5 cm and AB = 4.5 cm. If the area of quadrilateral ABCD is 30 cm^2 and DL is the altitude of ΔDAC, calculate the length DL.

Ans. In quadrilateral ABCD, diagonal AC divides it into two triangles ΔABC and ΔADC. In ΔABC, $\angle A = 90°$, BC = 7.5 cm and AB = 4.5 cm

But $BC^2 = AB^2 + AC^2$ (Pythagoras Theorem)

$\Rightarrow \quad (7.5)^2 = (4.5)^2 + AC^2 \Rightarrow 56.75 = 20.25 + AC^2$



$\Rightarrow \quad AC^2 = 56.25 - 20.25$

$\Rightarrow \quad AC^2 = 36 = (6)^2 \quad \therefore \quad AC = 6 \text{ cm}$

$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 4.5 \times 6 = 13.5 \text{ cm}^2$

Area of quadrilateral ABCD $= 30 \text{ cm}^2$

$\therefore \quad \text{Area of } \Delta ADC = (30 - 13.5) \text{ cm}^2 = 16.5 \text{ cm}^2$

$\text{Area of } \Delta ADC = \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times AC \times DL = \frac{1}{2} \times 6 \times DL = 3DL$

$\therefore \quad 3DL = 16.5 \Rightarrow DL = \frac{16.5}{3} = 5.5 \text{ cm}$

Q.26. The perimeter of a rectangle is 40.5 m and its breadth is 6 m. Find its length and area.

Ans. Let ABCD be a rectangle.

Let length be l m and breadth be 6 m

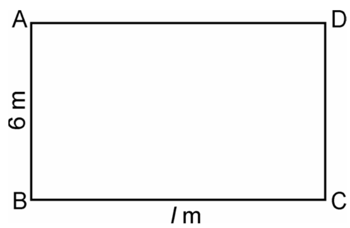
$\therefore \quad$ Perimeter of rectangle $= 40.5$ m

$2 \times (l + b) = 40.5 \text{ m}$

$\Rightarrow \quad 2 \times (l + 6) = 40.5 \text{ m} \Rightarrow 2l + 12 = 40.5$

$\Rightarrow \quad 2l = 40.5 - 12 \Rightarrow 2l = 28.5$

$\Rightarrow \quad l = \frac{28.5}{2} \Rightarrow l = 14.25 \text{ m}$



(ii) Area of rectangle ABCD $= l \times b = 14.25 \times 6 \text{ m}^2 = 85.50 \text{ m}^2 = 85.5 \text{ m}^2$.

Q.27. The perimeter of a rectangular field is $\frac{3}{5}$ km. If the length of the field is twice its width; find the area of the rectangle in sq. metres.

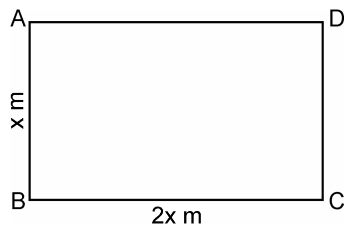
Ans. Let ABCD be a rectangular field.

Let breadth of rectangle $= x$ m and length $= 2x$ m

Perimeter of rectangular field $= \frac{3}{5}$ km

$2 \times (l + b) = \frac{3}{5} \times 1000 \text{ m}$

$\Rightarrow \quad 2 \times (2x + x) = 600 \Rightarrow 6x = 600$

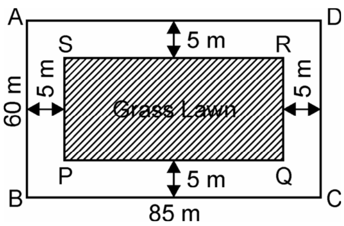


$\Rightarrow \quad x = \dfrac{600}{6} \Rightarrow x = 100$

$\therefore \quad l = 2x \text{ m} = 2 \times 100 \text{ m} = 200 \text{ m}$ and $b = x \text{ m} = 100 \text{ m}$

Area of rectangular field $= l \times b = 200 \times 100 \text{ m}^2 = 20000 \text{ m}^2$.

Q.28. A rectangular plot 85 m long and 60 m broad is to be covered with grass leaving 5 m all around. Find the area to be laid with grass.



Ans. Let ABCD be the plot and length of plot $= 85$ m and breadth $= 60$ m

Let PQRS be the grassy plot

Length of grassy lawn $= 85 \text{ m} - 2 \times 5 \text{ m}$

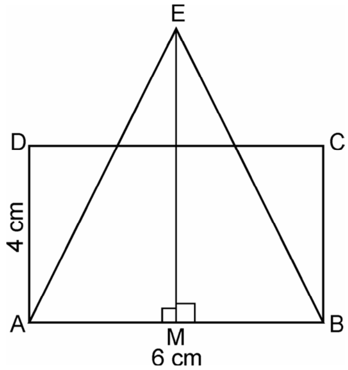
$= 85 \text{ m} - 10 \text{ m} = 75 \text{ m}$

Breadth of grassy lawn $= 60 \text{ m} - 2 \times 5 \text{ m}$

$= 60 \text{ m} - 10 \text{ m} = 50 \text{ m}$

Area of grassy lawn $= \text{length} \times \text{breadth} = (75 \times 50) \text{ m}^2 = 3750 \text{ m}^2$.

Q.29. The length and the breadth of rectangle are 6 cm and 4 cm respectively. Find the height of a triangle whose base is 6 cm and whose area is 3 times that of the rectangle.



Ans. Let ABCD be the rectangle

length of rectangle $= 6$ cm, breadth $= 4$ cm

Area of rectangle $= \text{length} \times \text{breadth}$

$= 6 \times 4 \text{ cm}^2 = 24 \text{ cm}^2$

In triangle EAB, base of triangle $= \text{AB} = 6$ cm

Let height of triangle $= \text{EM}$ cm

Area of triangle EAB $= 3 \times$ Area of rectangle ABCD

$\dfrac{1}{2} \times \text{base} \times \text{height} = 3 \times 24$

$\Rightarrow \quad \dfrac{1}{2} \times \text{AB} \times \text{EM} = 72 \Rightarrow \dfrac{1}{2} \times 6 \times \text{EM} = 72$

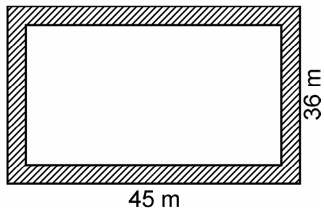
$\Rightarrow \quad 3\text{EM} = 72 \Rightarrow \text{EM} = 24$

$\therefore \quad$ Height of triangle $= 24$ cm

Q.30. A foot path of uniform width runs all around inside of a rectangular field 45 m long and 36 m wide. If the area of the path is 234 m^2. Find the width of the path.

Ans. Length of rectangular field $(l) = 45$ m

and breadth of the field $(b) = 36$ m



Let width of inner path $= x$ m

Thus, inner length of rectangular field $(l_1) = 45 - 2 \times x$

$= (45 - 2x)$ m

inner breadth of rectangular field $(b_1) = 36 - 2 \times x$

$= (36 - 2x)$ m

Area of path $= 234\ m^2$ (Given)

$\therefore \quad l \cdot b - l_1 \cdot b_1 = 234$

$\Rightarrow \quad 45 \times 36 - (45 - 2x)(36 - 2x) = 234$

$\Rightarrow \quad 1620 - (1620 - 90x - 72x + 4x^2) = 234$

$\Rightarrow \quad 1620 - (1620 - 162x + 4x^2) = 234$

$\Rightarrow \quad 1620 - 1620 + 162x - 4x^2 = 234 \Rightarrow -4x^2 + 162x = 234$

$\Rightarrow \quad 4x^2 - 162x + 234 = 0 \Rightarrow 2x^2 - 81x + 117 = 0$

$\Rightarrow \quad 2x^2 - 78x - 3x + 117 = 0$

$\Rightarrow \quad 2x(x - 39) - 3(x - 39) = 0$

$\Rightarrow \quad (2x - 3)(x - 39) = 0$

If $2x - 3 = 0$ then $x = \frac{3}{2}$

If $x - 39 = 0$, then $x = 39$ which is not possible.

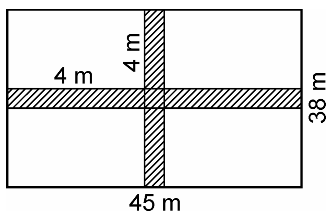
$\therefore \quad$ Width of path $= \frac{3}{2}$ m $= 1.5$ m

Q.31. The adjoining diagram shows two cross paths drawn inside a rectangular field 45 m long and 38 m wide, one parallel to length and the other parallel to breadth. The width of each path is 4 m. Find the cost of gravelling the paths at Rs 5.60 per m^2.

Ans. Length of rectangular field $(l) = 45$ m

Breadth of rectangular field $(b) = 38$ m

$\therefore \quad$ Width of path $(a) = 4$ m



$\therefore$ Area of path $= l\times a + b\times a - a\times a = (l+b)a - a^2$

$= (45+38)\times 4 - (4)^2\ \text{m}^2$

$= 83\times 4 - 16 = (332-16)\ \text{m}^2 = 316\ \text{m}^2$

Rate of gravelling the path $=$ Rs 5.60 per m^2

$\therefore$ Total cost of path $=$ Rs $316\times 5.60 =$ Rs 1769.60

Q32. A rectangle of area 144 cm^2 has its length equal to x cm. Write down its breadth in terms of x. Given that its perimeter is 52 cm, write down an equation in x and solve it to determine the dimensions of the rectangle.

Ans. Area of rectangle $= 144\ \text{cm}^2$ and perimeter of rectangle $= 52$ cm

Length of rectangle $= x$ cm

$\therefore$ Breadth of rectangle $= \dfrac{\text{Area of rectangle}}{\text{Length of rectangle}} = \dfrac{144}{x}$ cm

Breadth $= \dfrac{\text{Perimeter} - 2\text{Length}}{2}$

$= \dfrac{52-2x}{2} = (26-x)$ cm

Area of rectangle $=$ length $\times$ breadth

$\Rightarrow\ 144 = x\times(26-x) \Rightarrow 144 = 26x - x^2$

$\Rightarrow\ x^2 - 26x + 144 = 0$

$\Rightarrow\ x^2 - 18x - 8x + 144 = 0$

$\Rightarrow\ x(x-18) - 8(x-18) = 0 \Rightarrow (x-18)(x-8) = 0$

If $x-18=0$, then $x=18$

If $x-8=0$, then $x=8$

$\therefore\ x = 18$ or 8

Hence, length of rectangle $= 18$ cm and breadth of rectangle $= 8$ cm

Q.33. The perimeter of a rectangular plot is 130 m and its area is 1000 m^2. Take the length of the plot as x metres. Use the perimeter to write the value of breadth in terms of x. Use the values of length, breadth and area to write an equation in x. Solve the equation and calculate the length and breadth of the plot.

Ans. Area of rectangular field $= 1000\ \text{m}^2$ and perimeter of rectangular field $= 130$ m

Let length of the field $= x$ m

$\therefore$ Breadth of rectangular field $= \dfrac{130-2x}{2} = (65-x)$ m

$\therefore$ Area of rectangular field $= l \times b$

$\Rightarrow$ $1000 = x\,(65-x) \Rightarrow 1000 = 65x - x^2$

$\Rightarrow$ $x^2 - 65x + 1000 = 0 \Rightarrow x^2 - 40x - 25x + 1000 = 0$

$\Rightarrow$ $x\,(x-40) - 25\,(x-40) = 0$

$\Rightarrow$ $(x-40)\,(x-25) = 0$

If $x - 40 = 0$, then $x = 40$

If $x - 25 = 0$, then $x = 25$

$\therefore$ Length of the field $= 40$ m and breadth of the field $= 25$ m

Q.34. If the length of a rectangle is increased by 10 cm and the breadth decreased by 5 cm, the area remains unchanged. If the length is decreased by 5 cm and the breadth is increased by 4 cm, even then the area remains unchanged. Find the dimensions of the rectangle.

Ans. Let length of rectangle $= x$ cm and breadth of rectangle $= y$ cm

and area of rectangle $= l \times b = xy$ cm^2

Length of rectangle $= (x+10)$ cm

and breadth of rectangle $= (y-5)$ cm

$\therefore$ $(x+10)\,(y-5) = xy$

$\Rightarrow$ $xy - 5x + 10y - 50 = xy$

$\Rightarrow$ $-5x + 10y - 50 = 0 \Rightarrow x - 2y + 10 = 0$

$\Rightarrow$ $x - 2y = -10$ ...(i)

Length of rectangle $= x - 5$

and breadth of rectangle $= y + 4$

$\therefore$ $(x-5)\,(y+4) = xy \Rightarrow xy + 4x - 5y - 20 = xy$

$\Rightarrow$ $4x - 5y = 20$ ...(ii)

Multiply (i) by 5 and (ii) by 2, we get

$5x - 10y = -50$ ...(iii)

$8x - 10y = 40$ ...(iv)

Subtracting (iii) from (iv), we get

$$\begin{array}{r} 5x - 10y = -50 \\ \underline{8x - 10y = 40} \\ -3x = -90 \Rightarrow x = 30 \end{array}$$

Putting the value of x in (i), we get

$$30-2y=-10$$
$$\Rightarrow \quad -2y=-10-30=-40$$
$$\therefore \quad y=\frac{-40}{-2}=20$$

Hence, length of the rectangle $=30$ cm
and breadth of rectangle $=20$ cm

Q.35. A room is 13 m long and 9 m wide. Find the cost of carpeting the room with a carpet 75 cm wide and Rs 12.50 per meter.

Ans. Length of room $(l)=13$ m and width of room $(b)=9$ m

$$\therefore \quad \text{Area of room} = l\times b = 13 \text{ m}\times 9 \text{ m} = 117 \text{ m}^2$$
$$\text{Width of carpet} = 75 \text{ cm} = \frac{75}{100}=\frac{3}{4} \text{ m}$$
$$\therefore \quad \text{Length of carpet} = \text{Area} \div \text{Width} = 117 \div \frac{3}{4} = \frac{117\times 4}{3} \text{ m} = 156 \text{ m}$$

Rate of carpet $=$ Rs 12.50 per m

$\therefore$ Total cost $=$ Rs. $156\times$ Rs. $12.50=$ Rs. 1950.

Q.36. Find the area and perimeter of a square plot of land, and length of whose diagonal is 15 metres. Give your answer correct to 2 places of decimals.

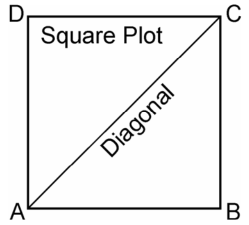
Ans. Let ABCD be the square plot of land. Let diagonal AC $=15$ m

or $\sqrt{2}\times\text{side}=15$ m

$$\text{Area of square} = (\text{side})^2 = \left(\frac{15\sqrt{2}}{2}\right)^2 = \frac{225}{2}\text{m}^2 = 112.5 \text{ m}^2$$
$$\text{Perimeter of square} = 4 \text{ side} = 4\times\frac{15\sqrt{2}}{2} = 30\sqrt{2} = 42.43 \text{ m}$$

Q.37. The area of square plot is 2025 m^2. Find the length of its side and the length of its diagonal.

Ans. Let ABCD be the square plot. AC be its diagonal



Area of square plot $=2025$ m^2

$$\text{Side or length of square plot} = \sqrt{\text{Area}}$$
$$= \sqrt{2025} \text{ m} = \sqrt{(5\times5)\times(9\times9)} \text{ m}$$
$$= 5\times 9 \text{ m} = 45 \text{ m}$$

$$\text{Diagonal AC} = \sqrt{2}\times\text{side}$$

$= \sqrt{2} \times 45 \text{ m} = 45 \times \sqrt{2} \text{ m}$

$= 45 \times 1.414 \text{ m} = 63.630 \text{ m} = 63.63 \text{ m}$

Q.38. The cost of cultivating a square field at the rate of 160 per hectare is Rs 1440. Find the cost of putting fence around it at the rate of 75 paise per metre.

Ans. Total cost = Rs 1440, Rate of cultivation = Rs 160 per hectare

$\therefore$ Area of the field $= \dfrac{1440}{160} = 9 \text{ hectare} = 90000 \text{ m}^2$

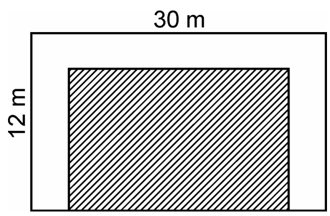
$\therefore$ Side of square field $= \sqrt{90000} = 300 \text{ m}$

Perimeter of the square field $= 4 \text{ side} = 4 \times 300 = 1200 \text{ m}$

Rate of fencing $= 75$ paise per metre

Hence, total cost $= \text{Rs}\dfrac{1200 \times 75}{100} = \text{Rs } 900$

Q.39. The shaded region of the given diagram represents the lawn in the form of a house. On the three sides of the lawn there are flowerbeds having a uniform width of 2 m.



(i) Fin d the length and the breadth of the lawn.

(ii) Hence, or other wise, find the area of the flower-beds.

Ans. Let BCDE is the lawn

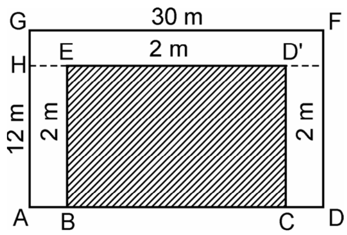
(i) Length of lawn BCDE $= BC$

$= AD - AB - CD = 30 \text{ m} - 2 \text{ m} - 2 \text{ m}$

$= 30 \text{ m} - 4 \text{ m} = 26 \text{ m}$

Breadth of lawn BCDE $= BE$

$= AG - GH = 12 \text{ m} - 2 \text{ m} = 10 \text{ m}$



(ii) Area of flower-beds

$=$ ar. (rectangle ADFG) $-$ ar. (lawn BCDE)

$= 30 \times 12 \text{ m}^2 - 26 \times 10 \text{ m}^2$

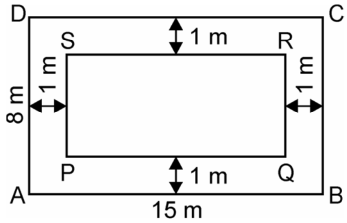
$= (360 - 260) \text{ m}^2 = 100 \text{ m}^2$

Q.40. A floor which measure 15 m × 8 m is to be laid with tiles measuring 50 cm × 25 cm. Find the number of tiles required.

Further, if a carpet is laid on the floor so that a space of 1 m exist between its edges and the edges of the floor, what fraction of the floor is uncovered.

Ans. Let rectangle ABCD be the floor and rectangle PQRS represents the carpet.

Length of floor $= 15$ m
Breadth of floor $= 8$ m



Area of floor $= 15 \times 8 \text{ m}^2 = 120 \text{ m}^2$

Length of tile $= 50 \text{ cm} = \frac{50}{100} \text{ m} = \frac{1}{2} \text{ m}$

breadth of tile $= 25 \text{ cm} = \frac{25}{100} \text{ cm} = \frac{1}{4} \text{ m}$

Area of 1 tile $= \frac{1}{2} \times \frac{1}{4} \text{ m}^2 = \frac{1}{8} \text{m}^2$

Number of tile required $= \frac{\text{Area of floor}}{\text{Area of 1 tile}} = \frac{120}{\frac{1}{8}} = 120 \times \frac{8}{1} = 960$

Length of carpet $= (15 - 2 \times 1) \text{ m} = (15 - 2) \text{ m} = 13 \text{ m}$
Breadth of carpet $= (8 - 2 \times 1) \text{ m} = (8 - 2) \text{ m} = 6 \text{ m}$

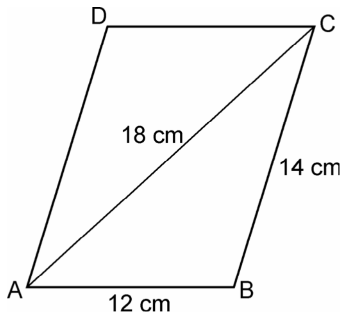
Area of carpet $= l \times b = 13 \times 6 \text{ m}^2 = 78 \text{ m}^2$
Area of floor uncovered $=$ Total area of floor $-$ Area of carpet
$= \text{ar. (rectangle ABCD)} - \text{ar. (rectangle PQRS)}$
$= (120 - 78) \text{ m}^2 = 42 \text{ m}^2$

Fraction of floor uncovered $= \frac{\text{Area of floor uncovered}}{\text{Total area of floor}} = \frac{42}{120} = \frac{7}{20}$

Q.41. Find the area of a parallelogram, if its two adjacent sides are 12 cm and 14 cm and if the diagonal connecting their ends is 18 cm.

Ans. $\because$ Diagonals bisect the parallelogram into two triangles of equal area.

$\therefore$ Area of parallelogram ABCD $= 2$ area of ΔABC
Sides of ΔABC are 12 cm, 14 cm and 18 cm



$\therefore \quad s = \frac{a+b+c}{2} = \frac{12+14+18}{2} = \frac{44}{2} = 22$

Area of $\Delta ABC = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$
$= \sqrt{22\,(22-12)\,(22-14)\,(22-18)}$
$= \sqrt{22 \times 10 \times 8 \times 4}$
$= \sqrt{11 \times 2 \times 2 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \sqrt{2^2 \times 2^2 \times 2^2 \times 110}$
$= 2 \times 2 \times 2\sqrt{110} = 8\sqrt{110} \text{ cm}^2$

Hence, area of parallelogram ABCD $= 2\times \text{ar.}(\Delta ABC) = 2\times 8\sqrt{110} = 16\sqrt{110}\ \text{cm}^2$

$= 16\times 10.488 = 167.81\ \text{cm}^2 = 167.8\ \text{cm}^2$

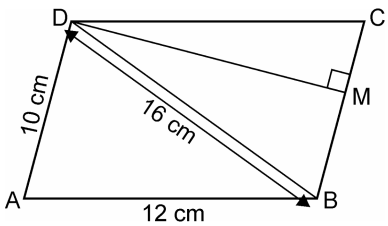
Q.42. Two adjacent sides of a parallelogram are 10 cm and 12 cm. If one diagonal of it is 16 cm long; find area of the parallelogram. Also, find distance between its shorter sides.

Ans. Let ABCD be the parallelogram with sides AD = 10 cm and side AB = 12 cm

Diagonal BD = 16 cm

Let DM be the distance between the shorter sides AD and BC

Area of || gm ABCD $= 2\times \text{ar.}(\Delta ABD)$



Area of ΔABD,

Let $a = 16$ cm, $b = 10$ cm and $c = 12$ cm

$$s = \frac{a+b+c}{2} = \frac{16+10+12}{2}\text{cm}$$

$$= \frac{38}{2}\ \text{cm} = 19\ \text{cm}$$

Using Hero's Formula

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{19(19-16)(19-10)(19-12)}\ \text{cm}^2$

$= \sqrt{19(3)(9)(7)}\ \text{cm}^2 = 3\sqrt{19(3)(7)}\ \text{cm}^2$

$= 3\sqrt{19\times 21}\ \text{cm}^2 = 3\sqrt{399}\ \text{cm}^2 = 3\times 19.97 = 59.91\ \text{cm}^2$

Area of || gm ABCD $= 2\times \text{ar.}(\Delta ABD) = 2\times 59.91\ \text{cm}^2$

$= 119.82\ \text{cm}^2 = 119.8\ \text{cm}^2$

(ii) Distance between the shorter sides $=$ DM

Area of || gm ABCD $= 119.82\ \text{cm}^2$

$\text{base}\times\text{height} = 119.82$

$BC\times DM = 119.82 \Rightarrow AD\times DM = 119.82$

$10\times DM = 119.82$

$$DM = \frac{119.82}{10} = 11.982 = 11.98$$

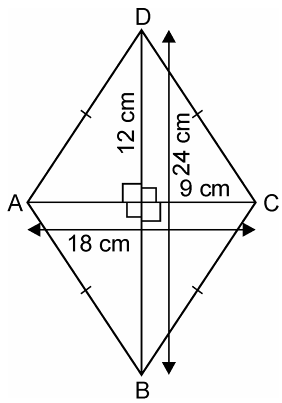
$\therefore$ Distance between shorter sides AD and BC $= 11.98$ cm

Q.43. The area of a rhombus is 216 square cm. If its one diagonal is 24 cm; find :

(i) length of its other diagonal.

(ii) length of its side.

(iii) perimeter of the rhombus.



Ans. Let ABCD be the rhombus then $AB = BC = CD = DA$

Diagonal $BD = 24$ cm

Diagonals AC and BD bisect each other at O at right angles.

$\therefore \quad DO = OB = \frac{24}{2} \text{cm} = 12 \text{ cm}$

(i) Area of rhombus $= 216$ sq. cm

$\frac{1}{2} \times \text{one diagonal} \times \text{second diagonal} = 216$ sq. cm

$\Rightarrow \quad \frac{1}{2} \times BD \times AC = 216 \Rightarrow \frac{1}{2} \times 24 \times AC = 216$

$\Rightarrow \quad AC = \frac{216}{12} \text{cm} = 18 \text{ cm}$

Hence, other diagonal $= 18$ cm

(ii) $AO = OC = \frac{18}{2} \text{ cm} = 9 \text{ cm}$

From triangle AOD in which $\angle AOD = 90°$

Using Pythagoras Theorem,

$AD^2 = AO^2 + OD^2 \Rightarrow AD^2 = (9)^2 + (12)^2$

$\Rightarrow \quad AD^2 = 81 + 144 = 225$

Taking square root of both sides, we get

$\Rightarrow \quad \sqrt{AD^2} = \sqrt{81+144}$

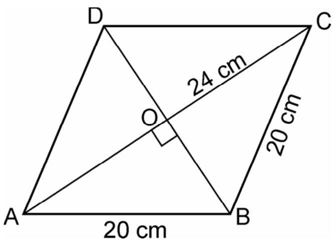
$\Rightarrow \quad \sqrt{AD \times AD} = \sqrt{225}$

$\Rightarrow \quad AD = \sqrt{15 \times 15} = 15$

$\therefore \quad$ Side of rhombus $= 15$ cm

(iii) Perimeter of rhombus ABCD $= 4 \times \text{side} = 4 \times 15 \text{ cm} = 60$ cm

Q.44. Find the area of a rhombus, one side of which measures 20 cm and one of whose diagonals is 24 cm.



Ans. In rhombus ABCD, each side $= 20 \text{ cm}$ and

One diagonal $AC = 24$ cm

$\because$ The diagonals of a rhombus bisect each other at right angles. $\therefore$ AC and BD bisect each other at O at right angle.

Hence, ΔAOB is a right angled triangle in which $AB = 20$ cm

$\therefore \quad AO = \frac{1}{2}AC = \frac{1}{2} \times 24 = 12 \text{ cm}$

$AB^2 = AO^2 + OB^2$ [Using Pythagoras Theorem]

$\Rightarrow \quad (20)^2 = (12)^2 + OB^2$

$\Rightarrow \quad 400 = 144 + BO^2$

$\Rightarrow \quad OB^2 = 400 - 144 = 256 = (16)^2$

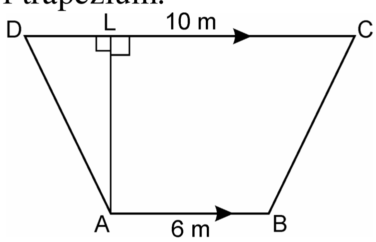
$\therefore \quad OB = 16 \text{ cm}$

$\therefore$ Diagonal BD $= 2 \times OB = 2 \times 16 = 32$ cm

Area of rhombus $= \dfrac{\text{Product of diagonals}}{2} = \dfrac{24 \times 32}{2} = 384 \text{ cm}^2$

Q.45. The cross-section of a canal is a trapezium in shape. If the canal is 10 m wide at the top, 6 m wide at the bottom and the area of the cross-section is 72 sq. m; determine its depth.

Ans. Let ABCD be the cross-section of canal in the shape of trapezium.



AB = 6 m, DC = 10 m

Let AL be the depth of canal

Area of cross-section $= 72$ sq. m

$\Rightarrow \quad \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{depth} = 72$

$\Rightarrow$

$\frac{1}{2} \times (AB + DC) \times AL = 72 \Rightarrow \frac{1}{2} \times (6 + 10) \times AL = 72$

$\Rightarrow \quad \frac{1}{2} \times (16) \times AL = 72 \Rightarrow 8 \times AL = 72$

$\Rightarrow \quad AL = \frac{72}{8} m = 9 \text{ m}$

Hence, depth of canal $= 9$ m

Q.46. Calculate the area of figure given below: which is not drawn to scale.

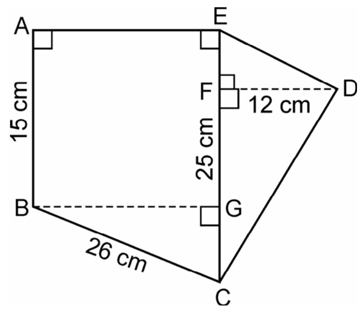
Ans. From figure,

$AB = 15$ cm, $BC = 26$ cm, $EC = 25$ cm, $DF = 12$ cm

Draw $BG \perp EC$

$\therefore \quad EG = AB = 15$ cm

$GC = EC - EG = 25 \text{ cm} - 15 \text{ cm} = 10$ cm



In ΔBGC, $\angle BGC = 90°$

Using Pythagoras Theorem

$$BG^2 = BC^2 - GC^2 = (26)^2 - (10)^2$$

$\Rightarrow \quad BG^2 = 676 - 100 \Rightarrow BG^2 = 576$

Taking square root of both sides, we get

$$\sqrt{BG^2} = \sqrt{576}$$

$$\sqrt{BG \times BG} = \sqrt{(8 \times 8) \times (3 \times 3)}$$

$$BG = 8 \times 3 \text{ cm} = 24 \text{ cm}$$

$\therefore \quad AE = BG = 24$ cm

$\therefore \quad$ Area of given figure $=$ Area of trapezium ABCE $+$ Area of ΔECD

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times (\text{height}) + \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (AB + EC) \times AE + \frac{1}{2} \times EC \times DF$$

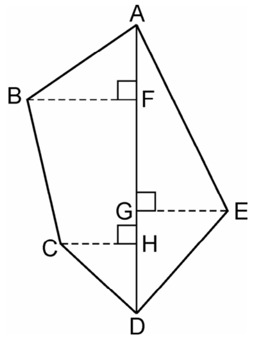
$$= \frac{1}{2} \times (15 + 25) \times 24 \text{ cm}^2 + \frac{1}{2} \times 25 \times 12 \text{ cm}^2$$

$$= \frac{1}{2} \times 40 \times 24 \text{ cm}^2 + \frac{1}{2} \times 25 \times 12 \text{ cm}^2$$

$$= 20 \times 24 \text{ cm}^2 + 25 \times 6 \text{ cm}^2$$

$$= 480 \text{ cm}^2 + 150 \text{ cm}^2 = 630 \text{ cm}^2$$

Q.47. The following diagram shows a pentagonal field ABCDE in which the lengths of AF, FG, GH and HD are 50 m, 40 m, 15 m and 25 m respectively; and the lengths of perpendiculars BF, CH and EG are 50 m, 25 m and 60 m respectively. Determine the area of the field.



Ans. ABCDE is a pentagonal field in which $AF = 50$ m

$FG = 40$ m, $GH = 15$ m

$HC = 25$ m, $BF = 50$ m, $HD = 25$ m

$GD = 40$ m, $CH = 25$ m

$EG = 60$ m

Area of pentagonal field ABCDE

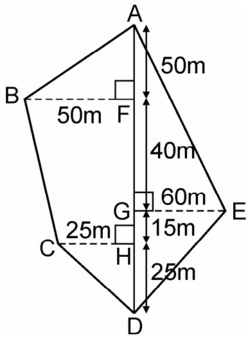
$= \text{ar. } (\Delta ABF) + \text{ar. (trapezium BCHF)} + \text{ar. } (\Delta CDH) + \text{ar. } (\Delta ADE)$

$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$+ \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AF \times BF + \frac{1}{2}(BF + CH)$

$\times FH + \frac{1}{2} \times HD \times CH + \frac{1}{2} \times AD \times EG$



$= \frac{1}{2} \times 50 \times 50 + \frac{1}{2} \times (50 + 25) \times (FG + GH)$

$+ \frac{1}{2} \times 25 \times 25 + \frac{1}{2} \times (AF + FG + GH + HD) \times 60$

$= 25 \times 50 \text{ m}^2 + \frac{1}{2} \times 75 \times (40 + 15) \text{ m}^2$

$+ \frac{1}{2} \times 625 \text{ m}^2 + \frac{1}{2} \times (50 + 40 + 15 + 25) \times 60 \text{ m}^2$

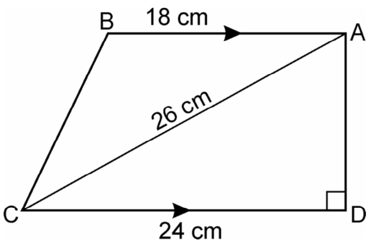
$= 1250 \text{ m}^2 + \frac{1}{2} \times 75 \times 55 \text{ m}^2 + 312.5 \text{ m}^2 + \frac{1}{2} \times (130) \times 60 \text{ m}^2$

$= 1250 \text{ m}^2 + \frac{1}{2} \times 4125 \text{ m}^2 + 312.5 \text{ m}^2 + 65 \times 60 \text{ m}^2$

$= 1250 \text{ m}^2 + 2062.5 \text{ m}^2 + 312.5 \text{ m}^2 + 3900 \text{ m}^2$

$= 7525.0 \text{ m}^2 = 7525 \text{ m}^2$

Q.48. The following diagram shows a trapezium ABCD in which AB || DC, $\angle D = 90°$, CD = 24 cm, AC = 26 cm and AB = 18 cm. Find the area of the trapezium.



Ans. In trapezium ABCD,

$\angle D = 90°$, AB || CD, AB = 18 cm, CD = 24 cm and AC = 26 cm

In right ΔADC,

$AC^2 = CD^2 + AD^2$ (Using Pythagoras Theorem)

$\Rightarrow (26)^2 = (24)^2 + AD^2 \Rightarrow 676 = 576 + AD^2$

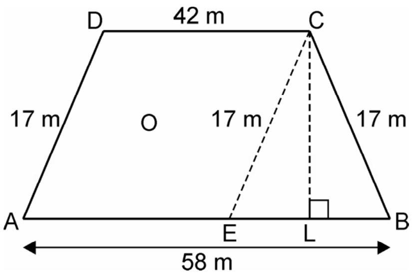
$\Rightarrow AD^2 = 676 - 576 = 100 = (10)^2$

$\therefore AD = 10$ cm

Area of trapezium $= \frac{1}{2}(AB + CD) \times AD = \frac{1}{2}(18 + 24) \times 10 \text{ cm}^2$

$= \frac{1}{2} \times 42 \times 10 = 210 \text{ cm}^2$

Q.49. The two parallel sides of a trapezium are 58 m and 42 m long. The other two sides are equal, each being 17 m. Find its area.



Ans. In trapezium ABCD, AB || DC

and AB = 58 m, CD = 42 m, BC = AD = 17 m

From C, draw CE || DA and $CL \perp AB$ meeting AB at E such that

AE = CD = 42 m and EB = AB − AE

$\Rightarrow EB = 58 - 42 = 16$ m

$CE = DA = 17$ m

$\because \Delta ECB$ is an isosceles triangle and $CL \perp EB$

$\therefore$ CL bisects EB at L

$\therefore EL = \frac{1}{2}EB = \frac{1}{2} \times 16 = 8$ m

In ΔCEL,

$CE^2 = CL^2 + EL^2$ (Using Pythagoras Theorem)

$\Rightarrow (17)^2 = CL^2 + (8)^2$

$\Rightarrow 289 = CL^2 + 64$

$\Rightarrow \quad CL^2 = 289 - 64 = 225 = (15)^2$

$\therefore \quad CL = 15 \text{ m}$

Area of trapezium ABCD $= \frac{1}{2} \times (AB + CD) \times CL = \frac{1}{2}(58 + 42) \times 15 \text{ m}^2$

$= \frac{1}{2} \times 100 \times 15 \text{ m}^2 = 750 \text{ m}^2$

Q.50. Find the circumference of the circle whose area is 16 times the area of the circle with diameter 1.4 m.

Ans. Radius of the small circle $= \frac{1.4}{2} = 0.7 \text{ m}$

$\therefore$ Area of circle $= \pi r^2 = \frac{22}{7} \times 0.7 \times 0.7 = 1.54 \text{ m}^2$

and area of bigger circle $= 1.54 \times 16 = 24.64 \text{ m}^2$

Let radius of bigger circle $= r$ m

$\therefore \quad \pi r^2 = 24.64$

$\Rightarrow \quad \frac{22}{7} r^2 = 24.64$

$\Rightarrow \quad r^2 = \frac{24.64 \times 7}{22} \Rightarrow r^2 = 1.12 \times 7 = 7.84$

$\Rightarrow \quad r = 2.8 \text{ m}$

$\therefore$ Circumference of circle $= 2\pi r = 2 \times \frac{22}{7} \times 2.8 = 17.6 \text{ m}$

Q.51. Calculate the circumference of a circle whose area is equal to the sum of areas of the circles with diameters 24 cm, 32 cm and 96 cm.

Ans. Radius of first circle $= \frac{24}{2} = 12 \text{ cm}$

$\therefore$ Area of first circle $= \pi r^2 = \frac{22}{7} \times 12 \times 12 \text{ cm}^2$

Similarly, area of second circle whose radius $= \frac{32}{2} = 16$ cm is $= \pi \times 16 \times 16$

$= \frac{22}{7} \times 16 \times 16 \text{ cm}^2$

Area of third circle whose radius is $\frac{96}{2} = 48$ cm

$$= \pi \times 48 \times 48 = \frac{22}{7} \times 48 \times 48 \text{ cm}^2$$

Area of bigger circle

$$= \frac{22}{7} \times 12 \times 12 + \frac{22}{7} \times 16 \times 16 + \frac{22}{7} \times 48 \times 48 \text{ cm}^2$$

$$= \frac{22}{7}(12 \times 12 + 16 \times 16 + 48 \times 48) \text{ cm}^2$$

$$= \frac{22}{7} \times (144 + 256 + 2304) \text{ cm}^2$$

$$= \frac{22}{7} \times 2704 \text{ cm}^2$$

Let radius of the bigger circle $= R$ cm

Area of bigger circle $= \pi R^2 = \frac{22}{7} \times 2704$

$$\Rightarrow \quad \frac{22}{7} R^2 = \frac{22}{7} \times 2704 \Rightarrow R^2 = 2704$$

$$\therefore \quad R = \sqrt{2704} = 52 \text{ cm}$$

Circumference of bigger circle $= 2\pi R = 2 \times \frac{22}{7} \times 52$ cm

$$= \frac{2288}{7} = 326.857 \text{ cm} = 326.86 \text{ cm}$$

Q.52. The ratio between the areas of two circles is 9 : 25. Find the ratio between :

(i) their radii, **(ii) their circumference**

Ans. Ratio between areas of two circles $= 9 : 25$

Let radius of first circle $= r_1$ and radius of second circle $= r_2$

$\therefore$ Area of first circle $= \pi r_1^2$ and area of second circle $= \pi r_2^2$

$$\therefore \quad \pi r_1^2 : \pi r_2^2 = 9 : 25$$

$$\Rightarrow \quad \frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{25} = \frac{(3)^2}{(5)^2}$$

$$\therefore \quad r_1 : r_2 = 3 : 5$$

(ii) Circumference of first circle $= 2\pi r_1$

and circumference of second circle $= 2\pi r_2$

$\therefore$ Ratio between their circumferences $= 2\pi r_1 : 2\pi r_2 = r_1 : r_2 = 3 : 5$

Q.53. The ratio between the circumferences of two circles is 4 : 9, find the ratio between their areas.

Ans. Let radius of first circle $= r_1$ and radius of second circle $= r_2$

$\therefore$ Ratio between their circumferences $= 2\pi r_1 : 2\pi r_2 = 4 : 9$

$$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{4}{9} \Rightarrow \frac{r_1}{r_2} = \frac{4}{9}$$

$\therefore$ $r_1 : r_2 = 4 : 9$

Area of first circle $= \pi r_1^2$ and Area of second circle $= \pi r_2^2$

$\therefore$ Ratio between areas of circles $\pi r_1^2 : \pi r_2^2$

$$= \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{(4)^2}{(9)^2} = \frac{16}{81}$$

Hence, ratio of areas of circles $= 16 : 81$

Q.54. The circumference of a circular garden is 572 m. Outside the garden a road, 3.5 m wide runs around it. Calculate the cost of repairing the road at the rate of Rs 375 per 100 sq. m.

Ans. Circumference of the garden $= 572$ m $=$ Circumference of circle

$\therefore$ Radius $(r) = \frac{572 \times 7}{2 \times 22} = 91$ m

Width of the road outside the garden $= 3.5$ m

$\therefore$ Outer radius $(R) = 91.0 + 3.5 = 94.5$ m

and area of the road $= \pi (R^2 - r^2) = \frac{22}{7}[(94.5)^2 - (91.0)^2]$ m^2

$$= \frac{22}{7}(94.5 + 91.0)(94.5 - 91.0)$$

$$= \frac{22}{7} \times 185.5 \times 3.5 \text{ m}^2 = 2040.5 \text{ m}^2$$

Cost of repairing the road at the rate of Rs 375 per 100 sq. m

$$= \frac{2040.5 \times 375}{100} = 7651.875 = \text{Rs } 7651.88 \text{ (approx.s)}$$

Q.55. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascents in 1 minute 28 seconds with a uniform speed of 1.1. m/s, calculate the number of complete revolutions the wheel makes in raising the bucket.

Ans. Diameter of wheel $(d) = 77$ cm Radius of wheel $= \frac{77}{2}$ cm

$\therefore$ Circumference of wheel $= 2\pi r = 2 \times \frac{22}{7} \times \frac{77}{2} = 242$ cm

Time taken by the bucket $= 1$ minutes 28 seconds $= 60 + 28 = 88$ secnds

Speed of rope $= 1.1$ m/s

$\therefore$ Length of rope $= 88 \times 1.1$ m $= 96.8$ m

$\therefore$ Number of revolutions $= \frac{96.8 \times 100}{242} = \frac{968 \times 100}{10 \times 242} = 40$

Q.56. The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km/hr. Given your answer, correct to the nearest km.

Ans. Diameter of wheel $(d) = 84$ cm

Radius of wheel $(r) = \frac{84}{2} = 42$ cm

$\therefore$ Circumference of wheel $= 2\pi r = 2 \times \frac{22}{7} \times 42 = 264$ cm

Distance covered in 1 sec $= 5 \times 264$ cm $= 1320$ cm

Distance covered in one hour $= 1320 \times 60 \times 60$ cm $= \frac{1320 \times 60 \times 60}{100 \times 1000}$ km

$= 47.52$ km/hr

Hence, Speed of wheel $= 47.52$ km/hr.

Q.57. A circular running track is the portion bounded by two concentric circles. If the cost of fixing the fence along the outer circumference of the track is Rs 3,696 at Rs 24 per meter and the cost of levelling the track is Rs 6,006 at Rs 12 per sq. m; calculate the width of the track.

Ans. Cost of fencing the outer track at the rate of Rs 24 per meter $=$ Rs 3696

$\therefore$ Circumference of outer track $= \frac{3696}{24} = 154$ m and radius of outer track $= \dfrac{\text{circumfernec of outer track}}{2\pi}$

$$= \frac{154}{2\times 22}\times 7 = \frac{49}{2}\text{ m}$$

Cost of levelling the track at the rate of Rs 12 per sq. m $=$ Rs 6006

$\therefore$ Area of the track $= \dfrac{6006}{12} = 500.5$ sq. m

Let the radius of inner circle $= r$

$\therefore$ Area of the track $= \pi(R^2 - r^2)$

$\Rightarrow \pi(R^2 - r^2) = 500.5 \Rightarrow \dfrac{22}{7}(R^2 - r^2) = \dfrac{5005}{10}$

$\Rightarrow R^2 - r^2 = \dfrac{5005}{10}\times\dfrac{7}{22} = 159.25$

$\Rightarrow \left(\dfrac{49}{2}\right)^2 - r^2 = 159.25$

$\Rightarrow 600.25 - r^2 = 159.25$

$\Rightarrow r^2 = 600.25 - 159.25 = 441 = (21)^2$

$\therefore r =$ radius of inner circle $= 21$ m

$\therefore$ Width of the track $= R - r = \dfrac{49}{2} - 21 = \dfrac{7}{2} = 3.5$ m

Q.58. The given figure shows a rectangle ABCD and a circle in it. Given AB = 10.5 cm and BC = 7 cm.

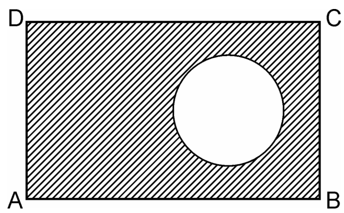
(i) If the area of shaded portion is 48.86 sq. cm; calculate the radius of the circle.

(ii) If the circumference of the circle is 15.4 cm; find the area of the shaded portion.

Ans. (i) Length AB of the rectangle $= 10.5$ cm and breadth of rectangle $= 7$ cm

$\therefore$ Area of the rectangle ABCD $= 10.5\times 7\text{ cm}^2 = 73.5\text{ cm}^2$

Area of shaded portion $= 48.86\text{ cm}^2$



$\therefore$ Area of circle $= (73.5 - 48.86)\text{ cm}^2 = 24.64$ sq. cm

Let radius of the circle $= r$ cm

$\therefore$ Area of circle $= \pi r^2 \Rightarrow \dfrac{22}{7}r^2 = 24.64$

$\Rightarrow \quad r^2 = 24.64 \times \frac{22}{7} = 7.84$

Hence, $r = \sqrt{7.84} = 2.8$ cm

(ii) Circumference of the circle $= 2\pi r = 15.4$ cm

$\therefore \quad$ Radius of circle $= \frac{C}{2\pi} = \frac{15.4 \times 7}{2 \times 22} = 2.45$ cm

$\therefore \quad$ Area of circle $= \pi r^2 = \frac{22}{7} \times 2.45 \times 2.45 = 18.865 \text{ cm}^2$

But area of rectangle $= 73.5 \text{ cm}^2$

$\therefore \quad$ Area of remaining part $= (73.5 - 18.865) \text{ cm}^2 = 54.635 \text{ cm}^2$

Q.59. The sum of the radii of two circles is 140 cm and the difference of their circumferences is 88 cm. Find the ratio between their area.

Ans. Difference of circumference of two circles $= 88$ cm

Sum of radii of two circles $= 140$ cm

Let radius of the first circle $= r$ cm

Then radius of the second circle $= (140 - r)$ cm

$\therefore \quad$ Circumference of first circle $= 2\pi r$

Circumference of second circle $= 2(140 - r) \times \pi$

According to the given problem, we get

$2\pi r - 2(140 - r)\pi = 88$

$\Rightarrow \quad 2\pi(r - 140 + r) = 88 \Rightarrow 2\pi(2r - 140) = 88$

$\Rightarrow \quad 2 \times \frac{22}{7}(2r - 140) = 88$

$\Rightarrow \quad 2r - 140 = \frac{88 \times 7}{2 \times 22} = 14$

$\Rightarrow \quad 2r = 14 + 140 = 154$

$\therefore \quad r = \frac{154}{2} = 77$ cm

$\therefore \quad$ Radius of first circle $= 77$ cm

and radius of second circle $= (140 - 77)$ cm $= 63$ cm

Area of first circle $= \pi r^2 = \pi (77)^2 \text{ cm}^2$

Area of second circle $= \pi (63)^2 \text{ cm}^2$

$\therefore$ Ratio of area of circles $= \dfrac{\pi (77)^2}{\pi (63)^2} = \dfrac{77 \times 77}{63 \times 63} = \dfrac{11 \times 11}{9 \times 9} = \dfrac{121}{81}$

Hence, ratio of areas of circles $= 121 : 81$.

Q.60. Find the area of a circle whose circumference is equal to the sum of the circumferences with diameters 36 cm and 20 cm.

Ans. Diameter of the first circle $= 36$ cm

$\therefore$ Radius of first circle $(r_1) = \dfrac{36}{2} = 18$ cm

and circumference of first circle $= 2\pi r = d\pi = 2 \times 18 \times \dfrac{22}{7} = 36 \times \dfrac{22}{7}$

Diameter of the second circle $= 20$ m

$\therefore$ Circumference of second circle $= 2\pi r = d\pi = 20 \times \dfrac{22}{7}$ cm

$\therefore$ Sum of the circumferences of two circles $= 36 \times \dfrac{22}{7} + 20 \times \dfrac{22}{7} = 56 \times \dfrac{22}{7} = 176$ cm

Circumference of the third circle $= 176$ cm

$\therefore$ Radius of third circle $= \dfrac{\text{Circumference of circle}}{2\pi} = \dfrac{176 \times 7}{2 \times 22} = 28$ cm

Area of third circle $= \pi r^2 = \dfrac{22}{7} \times 28 \times 28 \text{ cm}^2$

$= 22 \times 112 \text{ cm}^2 = 2464 \text{ cm}^2$

Q.61. A toothed wheel of radius 40 cm is attached to a smaller toothed wheel of diameter 24 cm. How many revolutions will the smaller wheel make when the larger one makes 150 revolutions.

Ans. Radius of bigger wheel $= 40$ cm.

$\therefore$ Circumference of circle $= 2\pi r = 2 \times \dfrac{22}{7} \times 40 = \dfrac{1760}{7}$ cm

Diameter of smaller wheel $= 24$ cm

Radius of wheel $= 12$ cm

$\therefore$ Circumference of wheel $= 2\pi r = 2 \times 12 \times \dfrac{22}{7}$ cm $= \dfrac{528}{7}$ cm

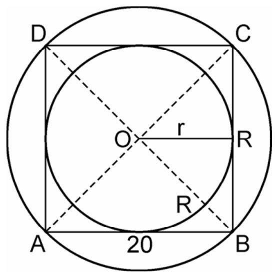
$\therefore$ Distance covered by larger wheel in 150 revolutions $= \dfrac{1760 \times 150}{7}$ cm

Number of revolutions made by the smaller wheel to cover $\frac{1760+150}{7}$ cm

$$= \frac{\text{Distance covered}}{\text{Circumference of smaller wheel}} = \frac{1760\times150\times7}{7\times528} = 500 \text{ revolutions.}$$

Q.62. The side of a square is 20 cm. Find the areas of the circumscribed and inscribed circles.

Ans. Side of the square $(a) = 20$ cm



$\therefore$ Radius of inscribed circle $(r) = \frac{1}{2}\times 20 = 10$ cm

$\therefore$ Area of inscribed circle $= \pi r^2 = \frac{22}{7}\times10\times10 \text{ cm}^2$

$$= \frac{2200}{7} \text{ cm}^2 = 314\frac{2}{7} \text{ cm}^2$$

Radius of the circumcircle $(R) = \frac{1}{2}$ diagonal of the square $= \frac{1}{2}(\sqrt{2}a)$

$$= \frac{\sqrt{2}}{2}a = \frac{\sqrt{2}}{2}\times 20 = 10\sqrt{2} \text{ cm}$$

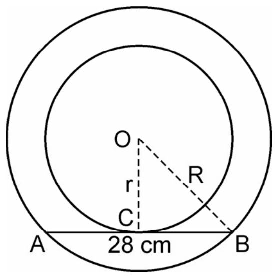
Thus, area of circumcircle $= \pi r^2 = \frac{22}{7}\times10\sqrt{2}\times10\sqrt{2} \text{ cm}^2$

$$= \frac{4400}{7} = 628\frac{4}{7} \text{ cm}^2$$

Q.63. Find the area of the region bounded by two concentric circles, if the length of the chord of the outer circle, touching the inner circle, is 28 cm.

Ans. Let the outer radius $= R$ and inner radius $= r$

and chord AB $= 28$ cm



$\because$ $OC \perp AB$, therefore C bisects AB

$\therefore$ $BC = \frac{28}{2} = 14$ cm

Now in ΔOCB,

$OB^2 - OC^2 = CB^2$ (Using Pythagoras Theorem)

$\Rightarrow$ $R^2 - r^2 = (14)^2 = 196$

Area of the region bounded by these two circles will be $= \pi(R^2 - r^2)$

$$= \frac{22}{7} \times 196 \text{ cm}^2 = 22 \times 28 \text{ cm}^2 = 616 \text{ cm}^2$$

Q.64. A sheet is 30 cm long and 10 cm wide. Circular pieces, all of equal diameters are cut from the sheet to prepare discs. Calculate the number of discs prepared if the diameter of each disc is :

(i) 1 cm (ii) 1.2 cm (iii) 1.5 cm (iv) 2.5 cm.

Ans. Length of sheet $(l) = 30$ cm and width of sheet $(b) = 10$ cm

$\therefore$ Area of the sheet $= l \times b$

$= 30 \times 10 = 300 \text{ cm}^2$

(i) Diameter of disc $= 1$ cm

Number of discs to be cut from length $= 30 \div 1 = 30$

and number of discs to be cut from width $= 10 \div 1 = 10$

Hence, total number of discs $= 30 \times 10 = 300$

(ii) Diameter of disc $= 1.2$ cm

$\therefore$ Number of discs to be cut from length $= 30 \div 1.2 = 25$

and number of discs to be cut from width $= 10 \div 1.2 = 8$

Thus total number of discs $= 25 \times 8 = 200$

(iii) Diameter of disc $= 1.5$ cm

Number of discs to be cut from length $= 30 \div 1.5 = 20$

and number of discs to be cut from width $= 10 \div 1.5 = 6$

$\therefore$ Total number of discs $= 20 \times 6 = 120$

(iv) Diameter of disc $= 2.5$ cm

Number of discs to be cut from length $= 30 \div 2.5 = 12$

and number of discs to be cut from width $= 10 \div 2.5 = 4$

$\therefore$ Total number of discs $= 12 \times 4 = 48$