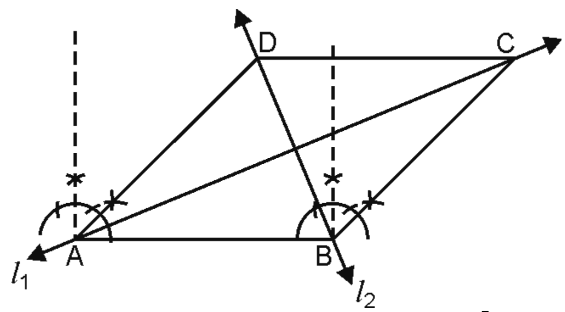
Question Bank
Symmetry Reflection, Loci and Constructions

1. Construct a rhombus ABCD with each side of length 5 cm and $\angle ABC = 135°$.
Draw its lines of symmetry.
Solution :



Draw a line segment AB = 5 cm. At B, draw $\angle ABC = 135°$ and cut off BC = 5 cm. At A, draw $\angle BAD = (180° – 135°) = 45°$ and cut off AD = 5 cm. Join DC. ABCD is the required rhombus. Join AC and extend it on both sides.

In ΔABC and ΔCDA, we have

$$AB = CD$$
$$BC = DA$$
$$\angle ABC = \angle CDA$$
$$\therefore \quad \Delta ABC \sim \Delta CDA \qquad [SAS]$$

Hence, AC is a line of symmetry of rhombus ABCD.
Similarly, join BD and BD is also a line of symmetry of rhombus ABCD.
Hence, AC and BD are two lines of symmetry of rhombus ABCD.

2. Use graph paper for this question. Plot the points A (– 2, 4), B (2, 1) and C(– 6, 1).

(i) Draw the line of symmetry of ΔABC.

(ii) Mark the point D, if the line in (i) and the line BC are both lines of symmetry of quadrilateral ABDC. Write down the co-ordinates of point D.

(iii) What kind of quadrilateral is figure ABDC?

(iv) Write down the equations of BC and the line of symmetry named in (i).

Solution :

First plot the points A(–2, 4), B(2, 1) and C(– 6, 1).

(i) $AC = \sqrt{(-6+2)^2 + (1-4)^2} = \sqrt{16+9} = 5$ units

$AB = \sqrt{(2+2)^2 + (1-4)^2} = \sqrt{16+9} = 5$ units

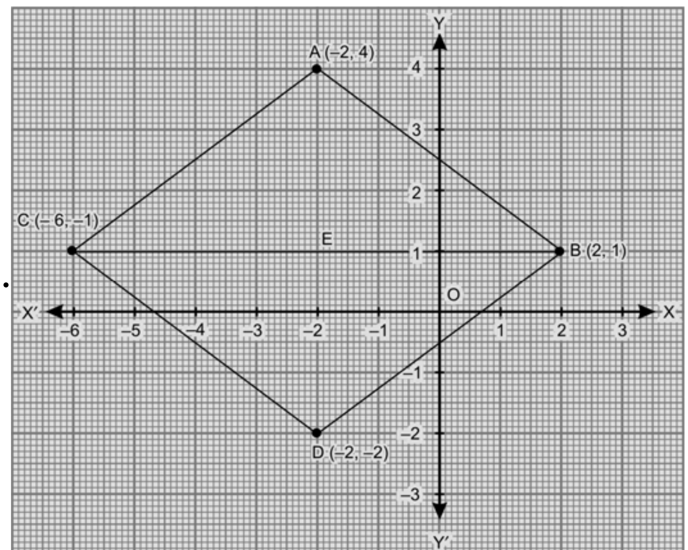
$BC = \sqrt{(-6+2)^2 + (1-1)^2} = \sqrt{64} = 8$ units

$\therefore$ ΔABC is isosceles.

Clearly E is the mid-point of BC.

So, AE is the bisector of $\angle A$.

$\therefore$ AE is the line of symmetry of ΔABC.



(ii) A is 3 units above from BC.

$\therefore$ Image of A in the line BC is point D which is 3 units below BC.

Clearly, co-ordinates of D are (2, –2)

(iii) AD = 6 units and BC = 8 units.

Thus, ABDC is a quadrilateral whose diagonals AD and BC are unequal and these diagonals are its lines of symmetry.

$\therefore$ ABDC is a rhombus.

(iv) BC is 1 unit above the *x*-axis.

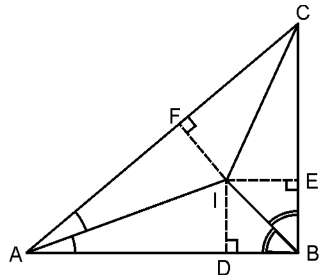
$\therefore$ Equation of BC is $y = 1$.

AD is 2 units away from y-axis on left hand side.

$\therefore$ Equation of AD is $x = -2$

3. Prove that the angle bisectors of a triangle pass through the same point.

Solution :



Given : A triangle ABC in which bisectors of $\angle A$ and $\angle B$ intersect at I.

To prove : IC bisects $\angle C$.

Construction : Join IC and draw $ID \perp AB$, $IE \perp BC$ and $IF \perp AC$.

Proof :

$\because$ I lies on bisector of $\angle A$.

$\therefore$ I is equidistant from AB and AC.

$\Rightarrow$ $ID = IF$ ...(i)

$\because$ I lies on bisector of $\angle B$.

$\therefore$ I is equidistant from AB and BC.

$\Rightarrow$ $ID = IE$ ...(ii)

From (i) and (ii), we have,

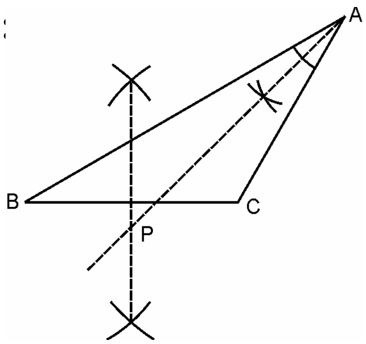
$IE = IF$

$\Rightarrow$ I is equidistant from CA and CB.

$\Rightarrow$ IC bisects $\angle C$. Proved.

4. In a triangle ABC, find a point P which is equidistant from AB and AC and also equidistant from B and C.

Solution :



Since point P is equidistant from AB and AC.

$\therefore$ P lies on the bisector of $\angle$A. ...(i)

Also, P is equidistant from B and C.

$\therefore$ P lies on the right bisector of BC. ...(ii)

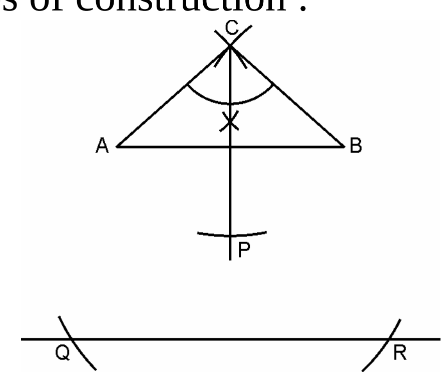
From (i) and (ii), we conclude that P is the point of intersection of bisector of $\angle$A and right bisector of BC.

So, to get the required point, draw bisector of $\angle$A and right bisector of BC.

The point of intersection of these two bisectors is the required point.

5. Construct an isosceles triangle ABC such that AB = 6 cm, BC = AC = 4 cm. Bisect $\angle$C internally and mark a point P on this bisector such that CP = 5 cm. Find points Q and R which are 5 cm from P and also 5 cm from the line AB.

Solution : Steps of construction :



(i) Draw a line segment AB = 6 cm.

(ii) With A as centre and radius equal to 4 cm, draw an arc.

(iii) With B as centre and radius equal to 4 cm, draw another arc, cutting the previous arc at C.

(iv) Join AC and BC.

(v) Draw angle bisector CP of $\angle C$ such that CP = 5 cm.

(vi) Draw a line parallel to AB at a distance of 5 cm.

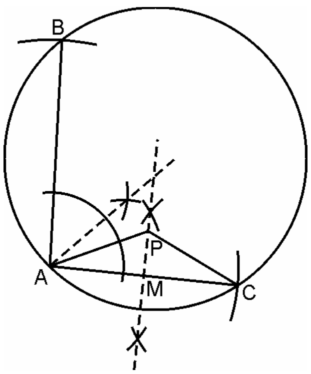
(vii) With P as centre and radius equal to 5 cm, draw two arcs, which cut the line drawn in (vi) at Q and R.
Q and R are the required points.

6. Using ruler and compasses only, draw a circle of radius 4 cm and mark two chords AB and AC of length 6 cm and 5 cm respectively.

(i) Construct the locus of points inside the circle that are equidistant from A and C. Prove your construction.

(ii) Construct the locus of points, inside the circle that are equidistant from AB and AC.

Solution : Steps of construction :



(i) Draw a circle of radius 4 cm.

(ii) Take any point A on the circumference of the circle. With A as centre and radius equal to 6 cm, draw an arc meeting the circle at B.

(iii) With A as centre and radius equal to 5 cm, draw an arc meeting the circle at C.

(iv) Join AB and AC.

(v) Draw the right bisector of AC.

(vi) Draw the bisector of $\angle BAC$.

Proof : Take any point P on the right bisector of AC. Join AP and PC.

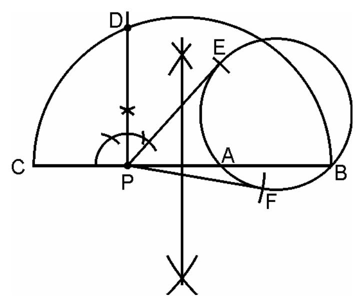
In ΔAMP and ΔCMP, we have

	AM = MC	[By construction]
	∠AMP = ∠CMP	[Each 90°]
	PM = PM	[Common]
∴	ΔAPM ≅ ΔCPM	[SAS]
⇒	AP = CP	[cpct]

7. Draw a circle of radius 2 cm. Take a point P outside it. Without using the centre, draw two tangents to the circle from the point P.

Solution :

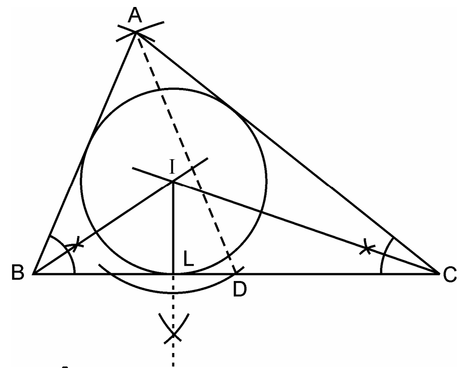
Steps of Construction :



1. Draw a circle of radius 2 cm.
2. Take a point P outside the circle and draw a secant PAB of the circle.
3. Produce BP to C such that PC = PA.
4. Draw a semi-circle with BC as a diameter.
5. Draw DP ⊥ CB, which cuts the semi-circle at D.
6. With P as centre and radius equal to PD, draw two arcs, which cut the circle at E and F.
7. Join PE and PF, which are the required tangents.

8. Construct a ΔABC in which AB = 4.5 cm, BC = 7 cm and median AD = 4 cm. Draw the inscribed circle of the triangle.

Solution :



Steps of Construction :

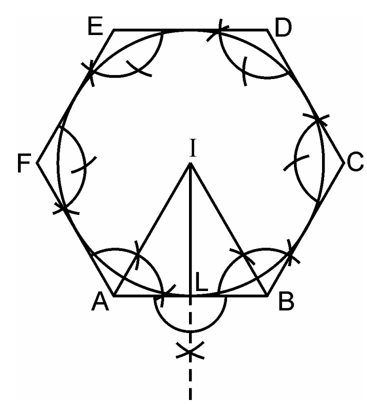
1. Draw BC = 7 cm and mark its mid-point as D.
2. With B as centre and radius equal to 4.5 cm, draw an arc.
3. With D as centre and radius equal to 4 cm, draw another arc, which cuts the previous arc at A.
4. Join AB, AD and AC.
   ABC is the required triangle.
5. Draw the angle bisectors of ∠B and ∠C, which meet at I.
6. From I, draw IL ⊥ BC.
7. With I as centre and radius equal to IL, draw a circle, which touches the three sides of the triangle.

9. Draw a regular hexagon of side 4 cm. Inscribe a circle in it.

Solution :

Steps of construction :

1. Draw AB = 4 cm.
2. Draw ∠ABC = 120° and BC = 4 cm.
3. Draw ∠BCD = 120° and CD = 4 cm.
4. Draw ∠CDE = 120° and DE = 4 cm.
5. Draw ∠DEF = 120° and EF = 4 cm.
6. Join AF. ABCDEF is the required hexagon.
7. Draw the bisector of ∠A and ∠B, which intersect each other at I.



8. Draw $IL \perp AB$.

9. With I as centre and radius as IL, draw
a circle which touches all the sides of the hexagon.

10. Points (3, 0) and (–1, 0) are invariant points under reflection in the line L_1; points (0, –3) and (0, 1) are invariant points on reflection in line L_2.

(i) Name the lines L_1 and L_2.

(ii) Write down the images of points P (3, 4) and Q (–5, –2) on reflection in L_1. Name the images as P′ and Q′ respectively.

(iii) Write down the images of P and Q on reflection in L_2. Name the images as P″ and Q″ respectively.

(iv) State or describe a single transformation that maps P′ onto P″.

Solution.

(i) We know that $R_x(x, y) = (x, -y)$

$\therefore$ $R_x(3, 0) = (3, 0)$ and $R_x(-1, 0) = (-1, 0)$

$\Rightarrow$ (3, 0) and (–1, 0) are invariant points on reflection in the x-axis.

Hence, L_1 is the x-axis.

We know that $R_y(x, y) = (-x, y)$ $\therefore$ $R_y(0, -3) = (0, -3)$ and $R_y(0, 1) = (0, 1)$

$\Rightarrow$ (0, –3) and (0, 1) are invariant points on reflection in the y-axis.

Hence, L_2 is the axis of y.

(ii) $R_x(x, y) = (x, -y)$

$\Rightarrow$ $R_x(3, 4) = (3, -4)$ and $R_x(-5, -2) = (-5, 2)$

Hence, P′ and Q′ are (3, – 4) and (–5, 2) respectively.

(iii) $R_y(x, y) = (-x, y)$

$\Rightarrow$ $R_y(3, 4) = (-3, 4)$ and $Ry(-5, -2) = (5, -2)$

Hence, P″ and Q″ are (–3, 4) and (5, –2) respectively.

(iv) We know that $R_o(x, y) = (-x, y)$

∴ Ro (3, – 4) = (–3, 4) = P′′

Hence, the single transformation that maps P′ to P′′ is the reflection in the origin.

11. Attempt this question on graph paper.

(i) Plot A (3, 2) and B (5, 4) on the graph paper. Take 10 small div. = 1 unit on both axes.

(ii) Reflect A and B in the *x*-axis to A′ and B′ respectively. Plot these on the same graph paper.

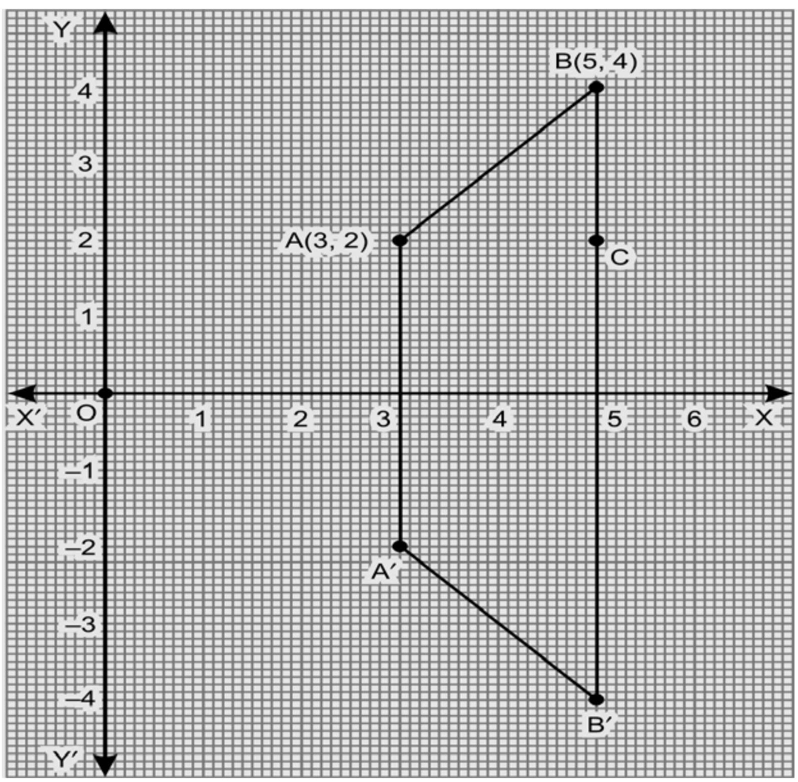
(iii) Write down

(a) the geometrical name of the figure ABB′A′

(b) the measure of the angle ABB′

(c) the image A′′ of A, when A is reflected in the origin

(d) the single transformation that maps A′ to A′′.



Solution.

(i) and **(ii)** See graph

(iii) (a) ABB′A′ is an isosceles trapezium.

(b) On the graph if AC be the perpendicular from A to BB′, then AC = 2, BC = 2 and ∠ACB = 90°, hence ∠ABC is angle of the isosceles right angled triangle ABC.

Hence, ABB′ = 45°.

(c) A″ is (–3, –2)

(d) A′ on the reflection in *y*-axis would be mapped at A″.

12. The point P (3, 4) is reflected to P′ in the *x*-axis and O′ is the image of O (the origin) when reflected in the line PP′. Using graph paper, give :

(i) The co-ordinates of P′ and O′.

(ii) The lengths of the segments PP′ and OO′.

(iii) The perimeter of the quadrilateral POP′O′.

(iv) The geometrical name of the figure POP′O′.

Solution.

(i) Since P′ is the image of P in the *x*-axis, the co-ordinate of P′ are (3, –4).

As O′ is the image of O (origin) in the line PP′, the co-ordinates of O′ are (6, 0).

(ii) Using graph, we find that length of segment PP′ = 8 units, length of segment OO′ = 6 units. .

(iii) OM = 3 units, MP = 4 units.

From right angled ΔOMP, by Pythagoras theorem, we get

$$OP^2 = OM^2 + MP^2 = 3^2 + 4^2 = 25$$

$$\Rightarrow \quad OP = 5 \text{ units}$$

Similarly, OP′ = O′P′ = O′P = 5 units.

∴ Perimeter of the quadrilateral POP′O′

= (5 + 5 + 5 + 5) units = 20 units.

(iv) POP′O′ is a rhombus.

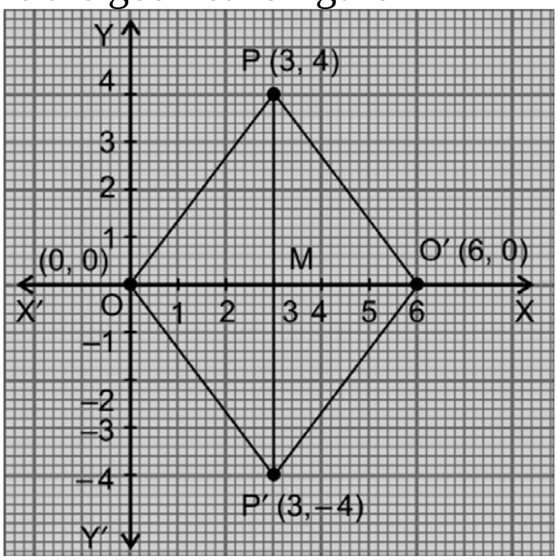
13. Use graph paper for this question.

(i) The point P(6, 5) is reflected in the line parallel to *y*-axis at a distance of 4 units on the positive side of the *x*-axis. P′ is the image. Plot it and find its coordinates.

(ii) The point P′ is mapped onto P″ on reflection in the *x*-axis.

(iii) P‴ is the image of P″ when reflected in the origin. Find the coordinates of P‴.

(iv) Name the geometric figure P′ P″ P‴ and find its area.



Solution.

(i) We know that the image of the point P(*x*, *y*) in the line parallel to *y*-axis at a distance of a from the *x*-axis (i.e., $x = a$) is the point P′ $(-x + 2a, y)$, if the line is taken on the positive side of *x*-axis.

∴ Coordinates of P′ are (–6 + 8, 5) i.e., (2, 5).

(ii) We know that $R_x(x, y) = (x, -y)$

∴ $R_x(2, 5) = (2, -5)$

i.e., coordinates of P″ are (2, –5).

(iii) We know that $R_o(x, y) = (-x, -y)$

∴ $R_o(2, -5) = (-2, 5)$

i.e., coordinates of P‴ are (–2, 5).

(iv) P′ P″ P‴ is a right angled triangle, right angled at P′.

Area of ΔP′ P″ P‴ = $\frac{1}{2} \times P' P'' \times P' P'''$

$= \frac{1}{2} \times 10 \times 4$ sq. units

= 20 sq. units.