1 RATIONAL AND IRRATIONAL NUMBERS

Q.1. Without actual division find which of the following rationals are terminating decimal:

(i) $\mathbf{\frac{9}{25}}$ **(ii)** $\mathbf{\frac{7}{12}}$ **(iii)** $\mathbf{\frac{121}{125}}$ **(iv)** $\mathbf{\frac{37}{78}}$

Ans. (i) In $\frac{9}{25}$, the prime factors of denominator 25 are 5, 5. Thus it is terminating decimal.

(ii) In $\frac{7}{12}$, the prime factors of denominator 12 are 2, 2 and 3. Thus it is not terminating decimal.

(iii) In $\frac{121}{125}$, the prime factors of denominator 125 are 5, 5 and 5. Thus it is terminating decimal.

(iv) In $\frac{37}{78}$, the prime factors of denominator 78 are 2, 3 and 13. Thus it is not terminating decimal.

Q.2. Represent each of the following as a decimal number.

(i) $\mathbf{\frac{4}{15}}$ **(ii)** $\mathbf{2\frac{5}{12}}$ **(iii)** $\mathbf{5\frac{31}{55}}$

Ans. (i) In $\frac{4}{15}$, using long division method:

$$\begin{array}{r} 0.266\ldots \\ 15\overline{)\,4.0000} \\ \underline{30} \\ 100 \\ \underline{90} \\ 100 \\ \underline{90} \\ \underline{10} \end{array}$$

Hence, $\frac{4}{15} = 0.266... = 0.2\overline{6}.$

(iii) In $2\frac{5}{12}$, using long division method:

$$\begin{array}{r|l} & 0.4166... \\ \hline 12 & 5.000 \\ & \underline{48} \\ & 20 \\ & \underline{12} \\ & 80 \\ & \underline{72} \\ & 80 \\ & \underline{72} \\ & 8 \end{array}$$

Hence, $2\frac{5}{12} = 2.4166... = 2.41\overline{6}$.

(iv) In $5\frac{31}{55}$, using long division method:

$$\begin{array}{r|l} & 0.5636363 \\ \hline 55 & 31.0000000 \\ & \underline{275} \\ & 350 \\ & \underline{330} \\ & 200 \\ & \underline{165} \\ & 350 \\ & \underline{330} \\ & 200 \\ & \underline{165} \\ & 350 \\ & \underline{330} \\ & 200 \\ & \underline{165} \\ & 35 \end{array}$$

Hence, $5\frac{31}{55} = 5.5636363... = 5.5\overline{63}$.

Q.3. Express each of the following as a rational number in the form of $\frac{p}{q}$, where $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.\overline{43}$ (iii) $0.2\overline{27}$ (iv) $0.2\overline{104}$

Ans. (i) Let $x = 0.\overline{6} = 0.6666$ ...(i)

Multiplying both sides of eqn. (i) by 10, we get

$10x = 6.6666$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$$\begin{array}{r} 10x = 6.6666 \\ x = 0.6666 \\ \hline 9x = 6 \end{array} \qquad \Rightarrow x = \frac{6}{9} = \frac{2}{3}$$

Hence, required fraction $= \frac{2}{3}$.

(ii) Let $x = 0.\overline{43} = 0.43434343$ ...(i)

Multiplying both sides of eqn. (i) by 100, we get

$100x = 43.434343$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$$\begin{array}{r} 100x = 43.434343 \\ x = \ \ 0.434343 \\ \hline 99x = 43 \end{array} \quad \Rightarrow 99x = 43 \Rightarrow x = \frac{43}{99}$$

Hence, required fraction $\frac{p}{q} = \frac{43}{99}$.

(iii) Let $x = 0.2\overline{27} = 0.2272727...$ ...(i)

Multiplying both sides of eqn. (i) by 10, we get

$10x = 2.272727...$ ...(ii)

Multiplying both sides of eqn. (ii) by 100, we get

$1000x = 227.272727...$ ...(iii)

Subtracting eqn. (ii) from (iii), we get

$$\begin{array}{r} 1000x = 227.272727... \\ 10x = \ \ \ 2.272727... \\ \hline 990x = 225 \end{array} \quad \Rightarrow 990x = 225 \Rightarrow x = \frac{225}{990} = \frac{5}{22}$$

Hence, required fraction $\frac{p}{q} = \frac{5}{22}$.

(iv) Let $x = 0.2\overline{104} = 0.2104104104...$ ...(i)

Multiplying both sides of eqn. (i) by 10, we get

$10x = 2.104104104...$ ...(ii)

Multiplying both sides of eqn. (ii) by 1000, we get

$10000x = 2104.104104104...$ ...(iii)

Subtracting eqn. (ii) from (iii), we get

$$\begin{array}{r} 10000x = 2104.104104104 \\ 10x = \ \ \ \ 2.104104104 \\ \hline 9990x = 2102 \end{array} \quad \Rightarrow \quad 9990x = 2102 \Rightarrow x = \frac{2102}{9990} = \frac{1051}{4995}$$

Hence, required fraction $= \frac{1051}{4995}$.

Q.4. Express each of the following as a vulgar fraction.

(i) $3.\overline{146}$ **(ii) $4.3\overline{24}$**

Ans. Let $x = 3.\overline{146} = 3.146146$ ...(i)

Multiplying both sides of eqn. (i) by 1000, we get

$$1000x = 3146.146146146$$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$$\begin{array}{r} 1000x = 3146.146146146 \\ x = 3.146146146 \\ \hline 999x = 3143 \end{array}$$

$$\Rightarrow 999x = 3143 \Rightarrow x = \frac{3143}{999}$$

Hence, required vulgar fraction $= \frac{3143}{999}$.

(ii) Let $x = 4.3\overline{24} = 4.324242424$ ...(i)

Multiplying both sides of eqn. (i) by 10, we get

$$10x = 43.24242424$$ ...(ii)

Multiplying both sides of eqn. (ii) by 100, we get

$$1000x = 4324.24242424$$ ...(iii)

Subtracting eqn. (ii) from eqn. (iii), we get

$$\begin{array}{r} 1000x = 4324.24242424 \\ 10x = \quad 43.24242424 \\ \hline 990x = 4281 \end{array}$$

$$\Rightarrow 990x = 4281 \Rightarrow x = \frac{4281}{990} = \frac{1427}{330}$$

Hence, required vulgar fraction $= \frac{1427}{330}$.

Q.5. Insert one rational number between:

(i) $\frac{3}{5}$ and $\frac{7}{9}$ **(ii) 8 and 8.04**

Ans. If a and b are two rational numbers, then between these two numbers, one rational number will be $\frac{(a+b)}{2}$.

Required rational number between $\frac{3}{5}$ and $\frac{7}{9}$

$$= \frac{1}{2}\left(\frac{3}{5}+\frac{7}{9}\right) = \frac{1}{2}\left(\frac{27+35}{45}\right) = \frac{1}{2}\times\frac{62}{45} = \frac{31}{45} \quad \therefore \quad \frac{3}{5} < \frac{31}{45} < \frac{7}{9}.$$

(ii) Required rational number between 8 and 8.04

$$=\frac{1}{2}(8+8.04)=\frac{1}{2}(16.04)=8.02 \quad \therefore \quad 8<8.02<8.04$$

Q.6. Insert two rational numbers between $\frac{3}{4}$ and $1\frac{1}{5}$

Ans. $\frac{3}{4}$ and $1\frac{1}{5} \Rightarrow \frac{3}{4}$ and $\frac{6}{5} \Rightarrow \frac{3}{4}<\frac{1}{2}\left(\frac{3}{4}+\frac{6}{5}\right)<\frac{6}{5} \Rightarrow \frac{3}{5}<\frac{1}{2}\left(\frac{15+24}{20}\right)<\frac{6}{5}$

$$\Rightarrow \frac{3}{4}<\frac{1}{2}\left(\frac{39}{20}\right)<\frac{6}{5} \Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left(\frac{39}{40}+\frac{6}{5}\right)<\frac{6}{5} \Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left(\frac{39+48}{40}\right)<\frac{6}{5}$$

$$\Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{1}{2}\left(\frac{87}{40}\right)<\frac{6}{5} \Rightarrow \frac{3}{4}<\frac{39}{40}<\frac{87}{80}<\frac{6}{5}$$

Hence, required rational numbers are $\frac{39}{40}$ and $\frac{87}{80}$.

Q.7. Insert three rational numbers between

(i) 4 and 5 **(ii) $\frac{1}{2}$ and $\frac{3}{5}$**

(iii) 4 and 4.5 **(iv) $2\frac{1}{3}$ and $3\frac{2}{3}$**

(v) $-\frac{1}{2}$ and $\frac{1}{3}$

Ans. (i) The given numbers are 4 and 5.

As, $4<5$

$$\Rightarrow 4<\frac{1}{2}\left(\frac{4+5}{1}\right)<5 \Rightarrow 4<\frac{9}{2}<5$$

$\Rightarrow 4<4.5<5$ ...(i)

Again, $4<\frac{1}{2}\left(4+\frac{9}{2}\right)<\frac{9}{2}$

$\Rightarrow 4<4.25<4.5$ ...(ii)

Again, $4.5<5 \Rightarrow 4.5<\frac{1}{2}(4.5+5)<5 \Rightarrow 4.5<4.75<5$ ...(iii)

$\therefore$ From eqn. (i), (ii) and (iii), we get $4<4.25<4.5<4.75<5$.

Thus, required rational numbers between 4 and 5 are 4.25, 4.75 and 4.5.

(ii) The given numbers are $\frac{1}{2}$ and $\frac{3}{5}$

As, $\frac{1}{2} < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{1}{2} + \frac{3}{5}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{5+6}{10}\right) < \frac{3}{5}$

$\Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{11}{10}\right) < \frac{3}{5} \quad \Rightarrow \frac{1}{2} < \frac{11}{20} < \frac{3}{5}$

Again, $\frac{1}{2} < \frac{1}{2}\left(\frac{1}{2} + \frac{11}{20}\right) < \frac{3}{5} \quad \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{21}{20}\right) < \frac{3}{5}$

$\frac{1}{2} < \frac{21}{40} < \frac{3}{5}$ ...(ii)

Again, $\frac{11}{20} < \frac{3}{5} \Rightarrow \frac{11}{20} < \frac{1}{2}\left(\frac{11}{20} + \frac{3}{5}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{23}{20}\right) < \frac{3}{5}$

$\Rightarrow \frac{1}{2} < \frac{23}{40} < \frac{3}{5}$ ...(iii)

From eqn. (i), (ii) and (iii), we get

$\frac{1}{2} < \frac{21}{40} < \frac{11}{20} < \frac{23}{40} < \frac{3}{15}$

Thus, required rational numbers between $\frac{1}{2}$ and $\frac{3}{5}$ are $\frac{21}{40}, \frac{11}{20}$ and $\frac{23}{40}$.

(iii) The given numbers are 4 and 4.5

As $4 < 4.5 \Rightarrow 4 < \frac{1}{2}(4 + 4.5) < 4.5$

$\Rightarrow 4 < 4.25 < 4.5$ ...(i)

$\Rightarrow 4 < \frac{1}{2}(4 + 4.25) < 4.25 \Rightarrow 4 < 4.125 < 4.25$ ...(ii)

Again, $4.25 < 4.5$

$\Rightarrow 4.25 < \frac{1}{2}(4.25 + 4.5) + 4.5 \Rightarrow 4.25 < 4.375 < 4.5$ ...(iii)

From eqn. (i), (ii) and (iii), we have $4 < 4.125 < 4.25 < 4.375 < 4.5$

Thus, required rational numbers between 4 and 4.5 are 4.125, 4.25 and 4.375.

(iv) The given numbers are $2\frac{1}{3}$ and $3\frac{2}{3}$ i.e., $\frac{7}{3}$ and $\frac{11}{3}$.

As $\frac{7}{3} < \frac{11}{3} \Rightarrow \frac{7}{3} < \frac{1}{2}\left(\frac{7}{3} + \frac{11}{3}\right) < \frac{11}{3}$

$\Rightarrow \frac{7}{3} < \frac{1}{2}\left(\frac{18}{3}\right) < \frac{11}{3} \Rightarrow \frac{7}{3} < \frac{18}{6} < \frac{11}{3}$

$\Rightarrow \frac{7}{3} < 3 < \frac{11}{3}$ ...(i)

Again, $\frac{7}{3} < \frac{1}{2}\left(\frac{7}{3} + \frac{3}{1}\right) < 3$

$\frac{7}{3} < \frac{8}{3} < 3$ ...(ii)

Again, $3 < \frac{11}{3}$

$3 < \frac{1}{2}\left(3 + \frac{11}{3}\right) < \frac{11}{3} \Rightarrow 3 < \frac{1}{2}\left(\frac{20}{3}\right) < \frac{11}{3}$

$3 < \frac{10}{3} < \frac{11}{3}$ ...(iii)

From eqn. (i), (ii) and (iii), we get

$\frac{7}{3} < \frac{8}{3} < 3 < \frac{10}{3} < \frac{11}{3}$.

Thus required rational numbers between $2\frac{1}{3}$ and $3\frac{2}{3}$ i.e., $\frac{7}{3}$ and $\frac{11}{3}$ are $\frac{8}{3}$, 3 and $\frac{10}{3}$.

Q.8. Find the decimal representation of $\frac{1}{7}$ and $\frac{2}{7}$. Deduce from the decimal representation of $\frac{1}{7}$, without actual calculation, the decimal representation of $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$.

Ans. Decimal representation of $\frac{1}{7}$ using long division method.

$$
\begin{array}{r}
0.142871 \\
7\overline{)1.000000} \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
10 \\
\underline{7} \\
3
\end{array}
$$

Thus decimal representation of $\frac{1}{7} = 0.\overline{142857}$

$\Rightarrow$ Decimal representation of $\frac{2}{7} = 2\times\frac{1}{7} = 2\times 0.\overline{142857} = 0.\overline{285714}$

$\Rightarrow$ Decimal representation of $\frac{3}{7} = 3\times\frac{1}{7} = 3\times 0.\overline{142857} = 0.\overline{428571}$

$\Rightarrow$ Decimal representation of $\frac{4}{7} = 4\times\frac{1}{7} = 4\times 0.\overline{142857} = 0.\overline{571428}$

$\Rightarrow$ Decimal representation of $\frac{5}{7} = 5\times\frac{1}{7} = 5\times 0.\overline{142857} = 0.\overline{714285}$

$\Rightarrow$ Decimal representation of $\frac{6}{7} = 6\times\frac{1}{7} = 6\times 0.\overline{142847} = 0.\overline{857142}$

Q.9. State, whether the following numbers are rational or irrational:

(i) $\left(2+\sqrt{2}\right)^2$ **(ii)** $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right)$

Ans. (i) $\left(2+\sqrt{2}\right)^2 = 4+2+2\times 2\times\sqrt{2} = 6+4\sqrt{2}$

Hence, it is an irrational number.

(iii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right) = (5)^2-\left(\sqrt{5}\right)^2$ [Using $(a+b)(a-b)=a^2-b^2$]

$= 25-5 = 20$

Hence, it is a rational number.

Q.10. Given universal set $= \{-6, -5\frac{3}{4}, -\sqrt{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, \sqrt{8}, 3.01, \pi, 8.47\}$

From the given set find:

(i) Set of rational numbers **(ii) Set of irrational numbers**

(iii) Set of integers **(iv) Set of non-negative integers**

Ans. The given universal set is

$$\{-6, -5\frac{3}{4}, -\sqrt{4}, -\frac{3}{5}, -\frac{3}{8}, 0, \frac{4}{5}, 1, 1\frac{2}{3}, \sqrt{8}, 3.01, \pi, 8.47\}$$

(i) Set of rational numbers

$$=\{-6,-5\frac{3}{4},-\sqrt{4},-\frac{3}{5},-\frac{3}{8},0,\frac{4}{5},1,1\frac{2}{3},3.01,8.47\}$$

(ii) Set of irrational numbers $=\{\sqrt{8},\pi\}$

(iii) Set of integers $=\{-6,-\sqrt{4},0,1\}$

(iv) Set of non-negative integers $=\{0,1\}$

Q.11. Use division method to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

Ans. $\sqrt{3}=1.73205...$

$$\begin{array}{r|l} & 1.73205 \\ \hline 1 & 3.00\ 00\ 00\ 00\ 00 \\ & 1 \\ \hline 27 & 200 \\ & 189 \\ \hline 343 & \ \ 1100 \\ & \ \ 1029 \\ \hline 3462 & \quad 7100 \\ & \quad 6924 \\ \hline 346405 & \quad\ 1760000 \\ & \quad\ 1732025 \\ \hline & \qquad\ 28975 \end{array}$$

It is non-terminating and non-recurring decimals.

$\therefore$ $\sqrt{3}$ is an irrational number.

$\sqrt{5}=2.2360679...$

$$\begin{array}{r|l} & 2.2360679 \\ \hline 2 & 5.00\ 00\ 00\ 00\ 00\ 00 \\ & 1 \\ \hline 42 & 100 \\ & \ \ 84 \\ \hline 443 & \ \ 1600 \\ & \ \ 1329 \\ \hline 4466 & \quad 27100 \\ & \quad 26796 \\ \hline 447206 & \quad\ 3040000 \\ & \quad\ 2683236 \\ \hline 4472127 & \qquad 35676400 \\ & \qquad 31304889 \\ \hline 44721349 & \qquad\ 437151100 \\ & \qquad\ 402492141 \\ \hline & \qquad\quad 34658959 \end{array}$$

It is non terminating and non recurring decimals.

$\therefore$ $\sqrt{5}$ is an irrational number.

Q.14. Show that $\sqrt{5}$ is not a rational number.

Ans. Let $\sqrt{5}$ is a rational number and let $\sqrt{5} = \frac{p}{q}$.

Where p and q have no common factor and $q \neq 0$.
Squaring both sides, we get

$$\left(\sqrt{5}\right)^2 = \left(\frac{p}{q}\right)^2 = 5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2 \qquad \text{...(i)}$$

$\Rightarrow$ p^2 is a multiple of 5 $\Rightarrow$ p is also multiple of 5
Let $p = 5m$ for some positive integer m.

$$p^2 = 25m^2 \qquad \text{...(ii)}$$

From eqn. (i) and (ii), we get

$$5q^2 = 25m^2 \Rightarrow q^2 = 5m^2 \therefore q^2 \text{ is multiple of 5} \Rightarrow q \text{ is multiple of 5}$$

Thus, p and q both are multiple of 5.
This shows that 5 is a common factor of p and q. This contradicts the hypothesis that p and q have no common factor, other than 1.
$\therefore$ $\sqrt{5}$ is not a rational number.

Q.13. Show that:

(i) $\left(\sqrt{3}+\sqrt{7}\right)$ is an irrational number

(ii) $\left(\sqrt{3}+\sqrt{5}\right)$ is an irrational number.

Ans. (i) Let $\left(\sqrt{3}+\sqrt{7}\right)$ is a rational number.

Then square of given number i.e., $\left(\sqrt{3}+\sqrt{7}\right)^2$ is rational.

$\Rightarrow$ $\left(\sqrt{3}+\sqrt{7}\right)^2$ is rational

$\Rightarrow$ $\left(\sqrt{3}\right)^2 + \left(\sqrt{7}\right)^2 + 2\sqrt{3}\times\sqrt{7} = 3+7+2\sqrt{21} = \left(10+2\sqrt{21}\right)$ is rational

But, $\left(10+2\sqrt{21}\right)$ being the sum of a rational and irrational is irrational. This contradiction arises by assuming that $\left(\sqrt{3}+\sqrt{7}\right)$ is rational number.

Hence, $\sqrt{3}+\sqrt{7}$ is an irrational number.

(ii) Let $\left(\sqrt{3}+\sqrt{5}\right)$ is a rational number.

Then square of given number i.e., $\left(\sqrt{3}+\sqrt{5}\right)^2$ is rational.

$\Rightarrow \quad \left(\sqrt{3}+\sqrt{5}\right)^2$ is rational

$\Rightarrow \quad \left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\sqrt{3}\times\sqrt{5}=3+5+2\sqrt{15}=\left(8+2\sqrt{15}\right)$ rational.

But, $\left(8+2\sqrt{15}\right)$ being the sum of a rational and irrational it is irrational. This contradiction arises by assuming that $\left(\sqrt{3}+\sqrt{5}\right)$ is rational.

Hence, $\left(\sqrt{3}+\sqrt{5}\right)$ is irrational number.

Q.14. Use method of contradiction to show that $\sqrt{3}$ is an irrational number.

Ans. (i) Now, Let $\sqrt{3}$ is a rational number

$$\sqrt{3}=\frac{p}{q} \qquad \text{(where } q\neq 0)$$

Then, $\left(\sqrt{3}\right)^2=\left(\frac{p}{q}\right)^2$

$\Rightarrow \quad 3=\frac{p^2}{q^2} \quad \Rightarrow \quad p^2=3q^2$

$\therefore \quad p^2$ is divisible by 3 as $3q^2$ is divisible by 3.

$\Rightarrow \quad p$ is divisible by 3. ...(i)

Let $p=3r$

Then $p^2=9r^2$ (On squaring both sides)

$\Rightarrow \quad 3q^2=9r^2 \quad \Rightarrow \quad q^2=3r^2$

$\because \quad 3r^2$ is also divisible by 3.

$\therefore \quad q$ is divisible by 3. ...(ii)

From (i) and (ii), we get

$\frac{p}{q}$ is divisible by 3.

$\therefore$ p and q have 3 as their common factor but $\frac{p}{q}$ is a rational number i.e. p and q have no common factor. $\therefore$ $\frac{p}{q}$ is not rational. So $\sqrt{3}$ is not rational. Hence, $\sqrt{3}$ is irrational number.

Q.15. Insert three irrational numbers between 0 and 1.

Ans. Three irrational numbers between 0 and 1 can be

$$0 < 0.1011001110001111... < 0.1010011000111... < 0.202002000200002... < 1$$

Q.16. Rationalise the denominator and simplify.

(i) $\frac{1}{3-\sqrt{5}}$ **(ii)** $\frac{6}{\sqrt{5}+\sqrt{2}}$

(iii) $\frac{1}{2\sqrt{5}-\sqrt{3}}$ **(iv)** $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

(v) $\frac{1}{1+\sqrt{5}+\sqrt{3}}$ **(vi)** $\frac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

Ans. (i) $\frac{1}{3-\sqrt{5}}$

Multiplying numerator and denominator by $3+\sqrt{5}$, we get

$$\frac{1}{3-\sqrt{5}}=\frac{3+\sqrt{5}}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$$

$$=\frac{3+\sqrt{5}}{(3)^2-\left(\sqrt{5}\right)^2}=\frac{3+\sqrt{5}}{9-5}=\frac{3+\sqrt{5}}{4} \qquad \{\because (a+b)(a-b)=a^2-b^2\}$$

(ii) $\frac{6}{\sqrt{5}+\sqrt{2}}$

Multiplying numerator and denominator by $\sqrt{5}-\sqrt{2}$, we get

$$\frac{6\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}=\frac{6\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2} \qquad \{\because\ (a+b)(a-b)=a^2-b^2\}$$

$$=\frac{6\left(\sqrt{5}-\sqrt{2}\right)}{5-2}=\frac{6\left(\sqrt{5}-\sqrt{2}\right)}{3}=2\left(\sqrt{5}-\sqrt{2}\right)$$

(iii) $\frac{1}{2\sqrt{5}-\sqrt{3}}$

Multiplying numerator and denominator by $2\sqrt{5}+\sqrt{3}$, we get

$$\frac{1}{2\sqrt{5}-\sqrt{3}}=\frac{\left(2\sqrt{5}+\sqrt{3}\right)}{\left(2\sqrt{5}-\sqrt{3}\right)\left(2\sqrt{5}+\sqrt{3}\right)}=\frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}=\frac{2\sqrt{5}+\sqrt{3}}{17}$$

(iv) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$$=\frac{(7+3\sqrt{5})\,(3-\sqrt{5})-(7-3\sqrt{5})\,(3+\sqrt{5})}{(3+\sqrt{5})\,(3-\sqrt{5})}$$

$$=\frac{21-7\sqrt{5}+9\sqrt{5}-15-21-7\sqrt{5}+9\sqrt{5}+15}{(3)^2-(\sqrt{5})^2}$$

$$=\frac{21+2\sqrt{5}-15-21+2\sqrt{5}+15}{9-5}=\frac{4\sqrt{5}}{4}=\sqrt{5}$$

(v) $\frac{1}{1+\sqrt{5}+\sqrt{3}}$

Multiplying numerator and denominator by $1-(\sqrt{5}+\sqrt{3})$, we get

$$\frac{1}{1+\sqrt{5}+\sqrt{3}}=\frac{1}{1+(\sqrt{5}+\sqrt{3})}\times\frac{1-(\sqrt{5}+\sqrt{3})}{1-(\sqrt{5}+\sqrt{3})}$$

$$=\frac{1-(\sqrt{5}+\sqrt{3})}{(1)^2-(\sqrt{5}+\sqrt{3})^2}=\frac{1-\sqrt{5}-\sqrt{3}}{1-(5+3+2\sqrt{15})}$$

$$=\frac{1-\sqrt{5}-\sqrt{3}}{1-8-2\sqrt{15}}=\frac{1-\sqrt{5}-\sqrt{3}}{-7-2\sqrt{15}}=\frac{\sqrt{5}+\sqrt{3}-1}{7+2\sqrt{15}}$$

Multiplying numerator and denominator by $7-2\sqrt{15}$, we get

$$\frac{\sqrt{5}+\sqrt{3}-1}{7+2\sqrt{15}}=\frac{\sqrt{5}+\sqrt{3}-1}{7+2\sqrt{15}}\times\frac{(7-2\sqrt{15})}{(7-2\sqrt{15})}$$

$$=\frac{7\sqrt{5}+7\sqrt{3}-7-2\sqrt{75}-2\sqrt{45}+2\sqrt{15}}{49-4\,(15)}$$

$$=\frac{7\sqrt{5}+7\sqrt{3}-7-2\times5\sqrt{3}-2\times3\sqrt{5}+2\sqrt{15}}{-11}$$

$$= \frac{7\sqrt{5}+7\sqrt{3}-7-10\sqrt{3}-6\sqrt{5}+2\sqrt{15}}{-11}$$

$$= \frac{\sqrt{5}-3\sqrt{3}-7+2\sqrt{15}}{-11} = \frac{-(7-\sqrt{5}+3\sqrt{3}-2\sqrt{15})}{-11}$$

$$= \frac{7-\sqrt{5}+3\sqrt{3}-2\sqrt{15}}{11}$$

(vi) $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

Multiplying numerator and denominator by $\sqrt{6}+\sqrt{5}+\sqrt{11}$, we get

$$\frac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} = \frac{[(\sqrt{6}+\sqrt{5})+\sqrt{11}]}{[(\sqrt{6}+\sqrt{5})-\sqrt{11}][\sqrt{6}+\sqrt{5})+\sqrt{11}]}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{(\sqrt{6}+\sqrt{5})^2-(\sqrt{11})^2} = \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{6+5+2\sqrt{30}-11}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}}$$

Multiplying numerator and denominator by $\sqrt{30}$, we get

$$= \frac{(\sqrt{6}+\sqrt{5}+\sqrt{11})\sqrt{30}}{2\sqrt{30}\times\sqrt{30}}$$

$$= \frac{\sqrt{180}+\sqrt{150}+\sqrt{330}}{2\times 30}$$

$$= \frac{\sqrt{36\times 5}+\sqrt{25\times 6}+\sqrt{330}}{60}$$

$$= \frac{6\sqrt{5}+5\sqrt{6}+\sqrt{330}}{60}$$

Q.17. If $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$, find the value of a and b.

Ans. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

Multiplying both sides numerator and denominator of L.H.S. by $(\sqrt{3}-1)$, we get

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)\,(\sqrt{3}-1)}{(\sqrt{3}+1)\,(\sqrt{3}-1)}$$

$$\Rightarrow \frac{3+1-2\sqrt{3}\times 1}{3-1}$$

$$\Rightarrow \frac{4-2\sqrt{3}}{2}$$

$$\Rightarrow 2-\sqrt{3}$$

But $\frac{\sqrt{3}-1}{\sqrt{3}+1}=a+b\sqrt{3}$, so $2-\sqrt{3}=a+b\sqrt{3}$

Comparing both sides

$\Rightarrow a=2$ and $b=-1$

Q.18. If $\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$, find the value of *a* and *b*.

Ans. $\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$

Multiplying numerator and denominator of L.H.S. by $3+\sqrt{2}$, we get

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}=\frac{(3+\sqrt{2})\,(3+\sqrt{2})}{(3-\sqrt{2})\,(3+\sqrt{2})}$$

$$\Rightarrow \frac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2}\Rightarrow \frac{9+2+2\times 3\sqrt{2}}{9-2}\Rightarrow \frac{11+6\sqrt{2}}{7}$$

$\Rightarrow \frac{11}{7}+\frac{6\sqrt{2}}{7}$ But $\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$, so $\frac{11}{7}+\frac{6\sqrt{2}}{7}=a+b\sqrt{2}$

Comparing both sides, $a=\frac{11}{7}$, $b=\frac{6}{7}$

Q.19. Simplify:

(i) $\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$ **(ii)** $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$ **(iii)** $\frac{19}{3\sqrt{2}-2\sqrt{3}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}$

Ans. (i) $\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$

By rationalising the denominator of each term, we get

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}=\frac{22}{2\sqrt{3}+1}\times\frac{2\sqrt{3}-1}{2\sqrt{3}-1}+\frac{17}{2\sqrt{3}-1}\times\frac{2\sqrt{3}+1}{2\sqrt{3}+1}$$

$$=\frac{44\sqrt{3}-22}{\left(2\sqrt{3}\right)^2-(1)^2}+\frac{34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{44\sqrt{3}-22}{12-1}+\frac{34\sqrt{3}+17}{12-1}=\frac{44\sqrt{3}-22}{11}+\frac{34\sqrt{3}+17}{11}$$

$$=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{11}=\frac{78\sqrt{3}-5}{11}$$

(ii) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

By rationalising the denominator of each term, we get

$$\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$=\frac{\sqrt{2\times6}+2}{(\sqrt{6})^2-(\sqrt{2})^2}-\frac{\sqrt{3\times6}-\sqrt{6}}{(\sqrt{6})^2-(\sqrt{2})^2}$$

$$=\frac{\sqrt{2\times2\times3}+2}{6-2}-\frac{\sqrt{3\times3\times2}-\sqrt{6}}{6-2}$$

$$=\frac{2\sqrt{3}+2-(3\sqrt{2}-\sqrt{6})}{4}=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

(iii) $\frac{18}{3\sqrt{2}-2\sqrt{3}}+\frac{1}{5\sqrt{2}+2\sqrt{3}}$

By rationalising the denominator of each term, we get

$$\frac{18}{3\sqrt{2}-2\sqrt{3}}+\frac{1}{5\sqrt{2}+2\sqrt{3}}=\frac{18}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{5\sqrt{2}+2\sqrt{3}}\times\frac{5\sqrt{2}-2\sqrt{3}}{5\sqrt{2}-2\sqrt{3}}$$

$$=\frac{54\sqrt{2}+36\sqrt{3}}{(3\sqrt{2})^2-(2\sqrt{3})^2}+\frac{5\sqrt{2}-2\sqrt{3}}{(5\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{54\sqrt{2}+36\sqrt{3}}{18-12}+\frac{5\sqrt{2}-2\sqrt{3}}{38}$$

$$=\frac{19(54\sqrt{2}+36\sqrt{3})+3(5\sqrt{2}-2\sqrt{3})}{114}$$

$$=\frac{1026\sqrt{2}+684\sqrt{3}+15\sqrt{2}-6\sqrt{3}}{114}$$

$$=\frac{1041\sqrt{2}+678\sqrt{3}}{114}=\frac{1041\sqrt{2}}{114}+\frac{678\sqrt{3}}{114}$$

$$=\frac{347\sqrt{2}}{38}+\frac{113\sqrt{3}}{19}$$

Q.20. If $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y=\frac{\sqrt{5}+2}{\sqrt{5}-2}$; find : x^2+y^2+xy

(i) x^2 (ii) y^2 (iii) xy (iv) x^2+y^2+xy

Ans. (i) $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}=\frac{\sqrt{5}-2}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$

$=\frac{(\sqrt{5}-2)^2}{5-4}=\frac{5+4-4\sqrt{5}}{1}=9-4\sqrt{5}\ \therefore\ x=9-4\sqrt{5}$

Squaring both sides, we get

$\Rightarrow\ x^2=(9-4\sqrt{5})^2=81+16(5)-72\sqrt{5}=81+80-72\sqrt{5}=161-72\sqrt{5}$

(ii) $y=\frac{\sqrt{5}+2}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{5+4+4\sqrt{5}}{5-4}=9+4\sqrt{5}$

$y=9+4\sqrt{5}$

Squaring both sides, we get

$\therefore\ y^2=(9+4\sqrt{5})^2=81+80+2\times9\times4\sqrt{5}=161+72\sqrt{5}$

(iii) $xy=(9-4\sqrt{5})\,(9+4\sqrt{5})=81-80=1$

(iv) $x^2+y^2+xy=161-72\sqrt{5}+161+72\sqrt{5}+1=323$

Q.21. Write down the values of:

(i) $\left(\frac{3}{2}\sqrt{2}\right)^2$ (i) $(5+\sqrt{3})^2$

(iii) $(\sqrt{6}-3)^2$ (iv) $(\sqrt{5}+\sqrt{6})^2$

Ans. (i) $\left(\frac{3}{2}\sqrt{2}\right)^2=\frac{3}{2}\sqrt{2}\times\frac{3}{2}\sqrt{2}=\frac{9}{4}(\sqrt{2})^2=\frac{9}{4}\times2=\frac{9}{2}$

(ii) $(5+\sqrt{3})^2=(5)^2+(\sqrt{3})^2+2(5)\,(\sqrt{3})$ [using $(a+b)^2=a^2+b^2+2ab$]

$=25+3+10\sqrt{3}$

$=28+10\sqrt{3}$

(iii) $(\sqrt{6}-3)^2=(\sqrt{6})^2+(3)^2-2\times\sqrt{6}\times3$ [using $(a-b)^2=a^2+b^2-2ab$]

$=6+9-6\sqrt{6}=15-6\sqrt{6}$

(iv) $(\sqrt{5}+\sqrt{6})=(\sqrt{5})^2+(\sqrt{6})^2+2\times\sqrt{5}\times\sqrt{6}$ [using $(a+b)^2=a^2+b^2+2ab$]

$=5+6+2\sqrt{30}=11+2\sqrt{30}$

Q.22. Rationalize the denominator of:

(i) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ **(ii)** $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

(iii) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **(iv)** $\dfrac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

Ans. (i) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Multiplying numerator and denominator by $\sqrt{3}-\sqrt{2}$, we get

$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{(\sqrt{3})^2+(\sqrt{2})^2-2\times\sqrt{3}\times\sqrt{2}}{3-2}$$

$$=\frac{3+2-2\sqrt{6}}{1}=5-2\sqrt{6}$$

(ii) $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

Multiplying numerator and denominator by $\sqrt{7}+\sqrt{5}$, we get

$$=\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{(\sqrt{7}+\sqrt{5})^2}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$=\frac{(\sqrt{7})^2+(\sqrt{5})^2+2\times\sqrt{7}\times\sqrt{5}}{7-5}$$

$$=\frac{7+5+2\sqrt{35}}{2}=\frac{12+2\sqrt{35}}{2}=\frac{2\,(6+\sqrt{35})}{2}$$

$$=6+\sqrt{35}$$

(iii) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Multiplying numerator and denominator by $\sqrt{5}+\sqrt{3}$, we get

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{(\sqrt{5})^2+(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}}{5-3}$$

$$= \frac{5+3+2\sqrt{15}}{2} = \frac{8+2\sqrt{15}}{2} = \frac{2\,(4+\sqrt{15})}{2} = 4+\sqrt{15}$$

(iv) $\dfrac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

Multiplying numerator and denominator by $2\sqrt{5}+3\sqrt{2}$, we get

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{(2\sqrt{5}+3\sqrt{2})^2}{(2\sqrt{5})^2-(3\sqrt{2})^2}$$

$$= \frac{(2\sqrt{5})^2+(3\sqrt{2})^2+2\times 2\sqrt{5}\times 3\sqrt{2}}{20-18}$$

$$= \frac{20+18+12\sqrt{10}}{2} = \frac{38+12\sqrt{10}}{2}$$

$$= \frac{2\,(19+6\sqrt{10})}{2} = 19+6\sqrt{10}$$

Q.23. Find the values of '*a*' and '*b*' in each of the following:

(i) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = a+b\sqrt{3}$ **(ii)** $\dfrac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

(iii) $\dfrac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3}-b\sqrt{2}$ **(iv)** $\dfrac{5+3\sqrt{2}}{5-3\sqrt{2}} = a+b\sqrt{2}$

(v) $\dfrac{\sqrt{3}+4\sqrt{2}}{3\sqrt{2}+5\sqrt{3}} = a-b\sqrt{3}$ **(vi)** $\dfrac{4\sqrt{5}+3\sqrt{2}}{3\sqrt{5}-2\sqrt{2}} = a+b\sqrt{10}$

Ans. (i) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = a+b\sqrt{3}$

Multiplying numerator and denominator of L.H.S. by $(2+\sqrt{3})$, we get

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{4-3}$$

$$= \frac{(2)^2+(\sqrt{3})^2+2\times 2\times\sqrt{3}}{1} \qquad \{\text{using } (a+b)^2 = a^2+2ab+b^2\}$$

$$= 4+3+4\sqrt{3} = 7+4\sqrt{3}$$

But $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = a+b\sqrt{3}$. So, $7+4\sqrt{3} = a+b\sqrt{3}$

Comparing both sides we get:

$a = 7$ and $b = 4$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$

Multiplying numerator and denominator of L.H.S. by $\sqrt{7}-2$, we get

$$\frac{\sqrt{7}-2}{\sqrt{7}+2}=\frac{\sqrt{7}-2}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}=\frac{(\sqrt{7}-2)^2}{7-4}$$

$$=\frac{(\sqrt{7})^2+(2)^2-2\times 2\times\sqrt{7}}{3} \qquad \{\text{using } (a-b)^2=a^2-2ab+b^2\}$$

$$=\frac{7+4-4\sqrt{7}}{3}=\frac{11-4\sqrt{7}}{3}$$

But $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b.$ So, $\frac{11}{3}-\frac{4\sqrt{7}}{3}=a\sqrt{7}+b.$

Comparing both sides, we get

$a\sqrt{7}=\frac{-4\sqrt{7}}{3}$ and $b=\frac{11}{3}$

$\Rightarrow$ $a=\frac{-4}{3}$ and $b=\frac{11}{3}$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}}=a\sqrt{3}-b\sqrt{2}$

Multiplying numerator and denominator of L.H.S. by $\sqrt{3}+\sqrt{2}$, we get

$$\frac{3}{\sqrt{3}-\sqrt{2}}=\frac{3}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{3\sqrt{3}+3\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3\sqrt{3}+3\sqrt{2}}{3-2}=\frac{3\sqrt{3}+3\sqrt{2}}{1}$$

Also $\frac{3}{\sqrt{3}-\sqrt{2}}=a\sqrt{3}-b\sqrt{2}.$ So, $3\sqrt{3}+3\sqrt{2}=a\sqrt{3}-b\sqrt{2}$

Comparing both sides, we get

$a=3$ and $b=-3$

(iv) $\dfrac{5+3\sqrt{2}}{5-3\sqrt{2}}=a+b\sqrt{2}$

Multiplying numerator and denominator of L.H.S. by $5+3\sqrt{2}$, we get

$$\frac{5+3\sqrt{2}}{5-3\sqrt{2}}=\frac{5+3\sqrt{2}}{5-3\sqrt{2}}\times\frac{5+3\sqrt{2}}{5+3\sqrt{2}}=\frac{(5+3\sqrt{2})^2}{(5)^2-(3\sqrt{2})^2}$$

$$=\frac{(5)^2+(3\sqrt{2})^2+2\times5\times3\sqrt{2}}{25-18}\qquad\{\text{using } (a+b)^2=a^2+2ab+b^2\}$$

$$=\frac{25+18+30\sqrt{2}}{7}=\frac{43+30\sqrt{2}}{7}$$

Also, $\dfrac{5+3\sqrt{2}}{5-3\sqrt{2}}=a+b\sqrt{2}$. So, $\dfrac{43}{7}+\dfrac{30\sqrt{2}}{7}=a+b\sqrt{2}$

Comparing both sides, we get :

$a=\dfrac{43}{7}$ and $b=\dfrac{30}{7}$

(v) $\dfrac{\sqrt{3}+4\sqrt{2}}{3\sqrt{2}+5\sqrt{3}}=a-b\sqrt{3}$

Multiplying numerator and denominator of L.H.S. by $3\sqrt{2}-5\sqrt{3}$, we get :

$$\frac{\sqrt{3}+4\sqrt{2}}{3\sqrt{2}+5\sqrt{3}}=\frac{\sqrt{3}+4\sqrt{2}}{3\sqrt{2}+5\sqrt{3}}\times\frac{3\sqrt{2}-5\sqrt{3}}{3\sqrt{2}-5\sqrt{3}}$$

$$=\frac{3\sqrt{2}\times\sqrt{3}-5\times3+12\times2-20\sqrt{2\times3}}{(3\sqrt{2})^2-(5\sqrt{3})^2}$$

$$=\frac{3\sqrt{2\times3}-15+24-20\sqrt{2\times3}}{18-75}$$

$$=\frac{9-17\sqrt{2\times3}}{-57}=-\frac{9}{57}+\frac{17\sqrt{6}}{57}=\frac{-3}{19}+\frac{17\sqrt{6}}{57}$$

But $\dfrac{\sqrt{3}+4\sqrt{2}}{3\sqrt{2}+5\sqrt{3}}=a-b\sqrt{3}$. So, $\dfrac{-3}{19}+\dfrac{17\sqrt{6}}{57}=a-b\sqrt{6}$

Comparing both sides, we get :

$a=-\dfrac{3}{19}$ and $b=\dfrac{-17}{57}$

(vi) $\dfrac{4\sqrt{5}+3\sqrt{2}}{3\sqrt{5}-2\sqrt{2}}=a+b\sqrt{10}$

Multiplying numerator and denominator of L.H.S. by $3\sqrt{5}+2\sqrt{2}$, we get

$$\frac{4\sqrt{5}+3\sqrt{2}}{3\sqrt{5}-2\sqrt{2}}=\frac{4\sqrt{5}+3\sqrt{2}}{3\sqrt{5}-2\sqrt{2}}\times\frac{3\sqrt{5}+2\sqrt{2}}{3\sqrt{5}+2\sqrt{2}}$$

$$=\frac{4\sqrt{5}\times3\sqrt{5}+4\sqrt{5}\times2\sqrt{2}+3\sqrt{2}\times3\sqrt{5}+3\sqrt{2}\times2\sqrt{2}}{(3\sqrt{5})^2-(2\sqrt{2})^2}$$

$$=\frac{12\times5+8\sqrt{10}+9\sqrt{10}+6\times2}{45-8}$$

$$=\frac{72+17\sqrt{10}}{37}=\frac{72}{37}+\frac{17\sqrt{10}}{37}$$

Also, $\dfrac{4\sqrt{5}+3\sqrt{2}}{3\sqrt{5}-2\sqrt{2}}=a+b\sqrt{10}$. So, $a+b\sqrt{10}=\dfrac{72}{37}+\dfrac{17\sqrt{10}}{37}$

Comparing both sides, we get :

$a=\dfrac{72}{37}$ and $b=\dfrac{17}{37}$