3 EXPANSION AND FACTORIZATION

Q.1. Using the standard formulae, expand each of the following

(i) $(\sqrt{2}m+\sqrt{3}n)^2$ **(ii)** $\left(2x+\frac{1}{3x}\right)^2$

(iii) $\left(\frac{2}{5}x+\frac{5}{6}y\right)^2$ **(iv)** $\left(\frac{1}{2}x-\frac{3}{2}y\right)^2$

(v) $\left(\frac{3a}{2b}-\frac{2b}{3a}\right)^2$ **(vi)** $\left(\frac{2x}{3}+\frac{3}{2y}-2\right)^2$

Ans. (i) $(\sqrt{2}m+\sqrt{3}n)^2=(\sqrt{2}m)^2+(\sqrt{3}n)^2+2\times\sqrt{2}m\times\sqrt{3}n$

$$=2m^2+3n^2+2\sqrt{6}mn$$

(ii) $\left(2x+\frac{1}{3x}\right)^2=(2x)^2+\left(\frac{1}{3x}\right)^2+2\times2x\times\frac{1}{3x}=4x^2+\frac{1}{9x^2}+\frac{4}{3}$

(iii) $\left(\frac{2}{5}x+\frac{5}{6}y\right)^2=\left(\frac{2}{5}x\right)^2+\left(\frac{5}{6}y\right)^2+2\times\frac{2}{5}x\times\frac{5}{6}y=\frac{4}{25}x^2+\frac{25}{36}y^2+\frac{2}{3}xy$

(iv) $\left(\frac{1}{2}x-\frac{3}{2}y\right)^2=\left(\frac{1}{2}x\right)^2+\left(\frac{3}{2}y\right)^2-2\times\frac{1}{2}x\times\frac{3}{2}y=\frac{1}{4}x^2+\frac{9}{4}y^2-\frac{3}{2}xy$

(v) $\left(\frac{3a}{2b}-\frac{2b}{3a}\right)^2=\left(\frac{3a}{2b}\right)^2+\left(\frac{2b}{3a}\right)^2-2\times\frac{3a}{2b}\times\frac{2b}{3a}=\frac{9a^2}{4b^2}+\frac{4b^2}{9a^2}-2$

(vi) $\left(\frac{2x}{3}+\frac{3}{2y}-2\right)^2=\left(\frac{2x}{3}\right)^2+\left(\frac{3}{2y}\right)^2+(2)^2+2\times\frac{2}{3}x\times\frac{3}{2y}-2\times\frac{3}{2y}\times2-2\times2\times\frac{2x}{3}$

$$=\frac{4}{9}x^2+\frac{9}{4y^2}+4+2\times\frac{x}{y}-\frac{6}{y}-\frac{8}{3}x$$

$$=\frac{4x^2}{9}+\frac{9}{4y^2}+4+\frac{2x}{y}-\frac{6}{y}-\frac{8x}{3}$$

Q.2. Without multiplying evaluate:

(i) $(101)^2$ (ii) $(502)^2$ (iii) $(97)^2$ (iv) $(998)^2$

Ans. (i) $(101)^2 = (100+1)^2 = (100)^2 + (1)^2 + 2\times100\times1$
$= 10000 + 1 + 200 = 10201$

(ii) $(502)^2 = (500+2)^2 = (500)^2 + (2)^2 + 2\times2\times500$
$= 250000 + 4 + 2000 = 252004$

(iii) $(97)^2 = (100-3)^2 = (100)^2 + (3)^2 - 2\times100\times3$
$= 10000 + 9 - 600 = 9409$

(iv) $(998)^2 = (1000-2)^2 = (1000)^2 + (2)^2 - 2\times1000\times2$
$= 1000000 + 4 - 4000 = 996004$

Q.3. Simplify:

(i) $\left(x+\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^2$ **(ii)** $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2 - \left(\frac{2b}{a}-\frac{a}{2b}\right)^2$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2 - \left(\frac{a}{2b}-\frac{2b}{a}\right)^2 - 4$ **(iv)** $(5a+3b)^2 - (5a-3b)^2 - 60ab$

Ans. (i) $\left(x+\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^2 = \left(x^2+\frac{1}{x^2}+2\right) + \left(x^2+\frac{1}{x^2}-2\right)$

$$= x^2 + \frac{1}{x^2} + 2 + x^2 + \frac{1}{x^2} - 2$$

$$= 2x^2 + \frac{2}{x^2} = 2\left(x^2+\frac{1}{x^2}\right)$$

(ii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2 - \left(\frac{2b}{a}-\frac{a}{2b}\right)^2$

$$= \left\{\left(\frac{a}{2b}\right)^2 + \left(\frac{2b}{a}\right)^2 + 2\times\frac{a}{2b}\times\frac{2b}{a}\right\} - \left\{\left(\frac{2b}{a}\right)^2 + \left(\frac{a}{2b}\right)^2 - 2\times\frac{2b}{a}\times\frac{a}{2b}\right\}$$

$$= \left(\frac{a^2}{4b^2}+\frac{4b^2}{a^2}+2\right) - \left(\frac{4b^2}{a^2}+\frac{a^2}{4b^2}-2\right)$$

$$= \frac{a^2}{4b^2} + \frac{4b^2}{a^2} + 2 - \frac{4b^2}{a^2} - \frac{a^2}{4b^2} + 2 = 4$$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)^2-\left(\frac{a}{2b}-\frac{2b}{a}\right)^2-4$

$$=\left[\left(\frac{a}{2b}\right)^2+\left(\frac{2b}{a}\right)^2+2\times\frac{a}{2b}\times\frac{2b}{a}\right]-\left[\left(\frac{a}{2b}\right)^2+\left(\frac{2b}{a}\right)^2-2\times\frac{a}{2b}\times\frac{2b}{a}\right]-4$$

$$=\left[\frac{a^2}{4b^2}+\frac{4b^2}{a^2}+2\right]-\left[\frac{a^2}{4b^2}+\frac{4b^2}{a^2}-2\right]-4$$

$$=\frac{a^2}{4b^2}+\frac{4b^2}{a^2}+2-\frac{a^2}{4b^2}-\frac{4b^2}{a^2}+2-4$$

$$=\frac{a^2}{4b^2}+\frac{4b^2}{a^2}+4-\frac{a^2}{4b^2}-\frac{4b^2}{a^2}-4=0$$

(iv) $(5a+3b)^2-(5a-3b)^2-60ab$

$$=\{(5a)^2+(3b)^2+2\times5a\times3b\}-\{(5a)^2+(3b)^2-2\times5a\times3b\}-60ab$$

$$=(25a^2+9b^2+30ab)-(25a^2+9b^2-30ab)-60ab$$

$$=25a^2+9b^2+30ab-25a^2-9b^2+30ab-60ab=0$$

Q.4. **(i) If $a+b=7$ and $ab=10$, find the value of $(a-b)$.**

(ii) If $x-y=5$ and $xy=24$, find the value of $(x+y)$.

Ans. (i) $(a-b)^2=(a+b)^2-4ab$

$$=(7)^2-4\times10=49-40=9=(\pm3)^2 \quad \therefore \quad (a-b)=\pm3$$

(ii) $(x+y)^2=(x-y)^2+4xy$

$$=(5)^2+4\times24=25+96=121=(\pm11)^2 \quad \therefore \quad x+y=\pm11$$

Q.5. **If $(3a+4b)=16$ and $ab=4$, find the value of $(9a^2+16b^2)$.**

Ans. $(3a+4b)=16$

Squaring both sides, we get

$(3a)^2+(4b)^2+2\times3a\times4b=(16)^2$

$\Rightarrow\ 9a^2+16b^2+24ab=256 \Rightarrow 9a^2+16b^2+24\times4=256$

$\Rightarrow\ 9a^2+16b^2+96=256 \Rightarrow 9a^2+16b^2=256-96=160$

Hence, $9a^2+16b^2=160$

Q.6. If $(a-b)=0.9$ and $ab=0.36$, find the values of (i) $(a+b)$ (ii) (a^2-b^2)

Ans. $a-b=0.9$ and $ab=0.36$

(i) $(a+b)^2=(a-b)^2+4ab$ ...(i)

Putting values of $(a+b)$ and $(a-b)$ in eqn. (i), we get

$$=(0.9)^2+4\times0.36=0.81+1.44=2.25=(\pm1.5)^2$$

$\therefore\ a+b=\pm1.5$

(ii) $a^2-b^2=(a+b)(a-b)$ ...(ii)

Putting values of $(a+b)$ and $(a-b)$ in eqn. (ii), we get

$$=\pm1.5\times0.9=\pm1.35$$

Q.7. If $x+y=\frac{7}{2}$ and $xy=\frac{5}{2}$; find: (i) $x-y$ (ii) x^2-y^2

Ans. (i) $x+y=\frac{7}{2},\ xy=\frac{5}{2}$

We know that $(x-y)^2=(x+y)^2-4xy$ ...(i)

Putting the values of $(x+y)$ and (xy) in eqn. (i), we get

$$(x-y)^2=\left[\frac{7}{2}\right]^2-4\times\frac{5}{2}\Rightarrow(x-y)^2=\frac{49}{4}-2\times5$$

$$\Rightarrow\ (x-y)^2=\frac{49}{4}-10\Rightarrow(x-y)^2=\frac{49-40}{4}\Rightarrow(x-y)^2=\frac{9}{4}$$

Taking square root of both sides, we get

$$\sqrt{(x-y)^2}=\sqrt{\frac{9}{4}}\quad\Rightarrow\quad\sqrt{(x-y)^2}=\pm\sqrt{\frac{3\times3}{2\times2}}\quad\Rightarrow\quad x-y=\pm\frac{3}{2}$$

(ii) $x^2-y^2=(x+y)\,(x-y)=\left(\frac{7}{2}\right)\left(\pm\frac{3}{2}\right)=\pm\frac{21}{4}$

Q.8. If $a+\frac{1}{a}=6$; find:

(i) $a-\frac{1}{a}$ **(ii)** $a^2-\frac{1}{a^2}$

Ans. (i) $a+\frac{1}{a}=6$

We know that $\left(a+\frac{1}{a}\right)^2-\left(a-\frac{1}{a}\right)^2=4$

Putting the value of $\left(a+\frac{1}{a}\right)$, we get

$$(6)^2-\left(a-\frac{1}{a}\right)=4 \Rightarrow -\left(a-\frac{1}{a}\right)^2=4-36 \Rightarrow \left(a-\frac{1}{a}\right)^2=32$$

Taking squaring root of both sides, we get

$$\sqrt{\left(a-\frac{1}{a}\right)^2}=\pm\sqrt{32} \Rightarrow \sqrt{\left(a-\frac{1}{a}\right)^2}=\pm\sqrt{16\times 2} \Rightarrow a-\frac{1}{a}=\pm 4\sqrt{2}$$

(ii) $a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$ Here $\left(a+\frac{1}{a}\right)=6$, $\left(a-\frac{1}{a}\right)=\pm 4\sqrt{2}$

$$a^2-\frac{1}{a^2}=(6)(\pm 4\sqrt{2})=\pm 24\sqrt{2}$$

Q.9. If $a=\frac{1}{(a-5)}$ where $a\neq 5$ and $a\neq 0$, find the values of:

(i) $\left(a-\frac{1}{a}\right)$ **(ii)** $\left(a+\frac{1}{a}\right)$

(iii) $\left(a^2-\frac{1}{a^2}\right)$ **(iv)** $\left(a^2+\frac{1}{a^2}\right)$

Ans. $a=\frac{1}{a-5} \Rightarrow a-5=\frac{1}{a}$

$\Rightarrow a-\frac{1}{a}=5$

(i) Hence, $a-\frac{1}{a}=5$

(ii) $\left(a+\frac{1}{a}\right)^2=\left(a-\frac{1}{a}\right)^2+4$

$$= (5)^2 + 4$$
$$= 25 + 4 = 29$$

Hence, $a + \frac{1}{a} = \pm\sqrt{29}$

(iii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= \pm\sqrt{29} \times 5$$
$$= \pm 5\sqrt{29}$$

(iv) $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2 = (5)^2 + 2 = 25 + 2 = 27$

Hence, $a^2 + \frac{1}{a^2} = \sqrt{27} = \sqrt{9 \times 3} = \pm 3\sqrt{3}$

Q.10. If $\left(x - \frac{1}{x}\right) = 5$, find the value of $\left(x^3 - \frac{1}{x^3}\right)$.

Ans. $x - \frac{1}{x} = 5$ (Given)

Cubing both sides, we get

$$\left(x - \frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = 125 \Rightarrow x^3 - \frac{1}{x^3} - 3 \times 5 = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 15 = 125 \Rightarrow x^3 - \frac{1}{x^3} = 125 + 15 = 140$$

Hence, $x^3 - \frac{1}{x^3} = 140$

Q.11. If $\left(x+\frac{1}{x}\right)=4$, find the values of:

(i) $\left(x^3+\frac{1}{x^3}\right)$ **(ii)** $\left(x-\frac{1}{x}\right)$

Ans. $x+\frac{1}{x}=4$

Cubing both sides, we get

(i) $\left(x+\frac{1}{x}\right)^3=(4)^3$

$\Rightarrow \quad x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=64 \Rightarrow x^3+\frac{1}{x^3}+3\times4=64$

$\Rightarrow \quad x^3+\frac{1}{x^3}+12=64 \Rightarrow x^3+\frac{1}{x^3}=64-12=52$

Hence, $x^3+\frac{1}{x^3}=52$

(ii) $\left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4=(4)^2-4=16-4=12$

$\Rightarrow \quad x-\frac{1}{x}=\pm\sqrt{12}=\pm\sqrt{4\times3}$

$\Rightarrow \quad x-\frac{1}{x}=\pm2\sqrt{3}$

Q.12. If $\left(a-\frac{1}{a}\right)=\sqrt{5}$, Find the values of: (i) $\left(a+\frac{1}{a}\right)$ (ii) $\left(a^3+\frac{1}{a^3}\right)$

Ans. $a^2-\frac{1}{a^2}=\sqrt{5}$

(i) $\left(a+\frac{1}{a}\right)^2=\left(a-\frac{1}{a}\right)^2+4=(\sqrt{5})^2+4=5+4=9$

$\therefore \quad a+\frac{1}{a}=\pm\sqrt{9}=\pm3$

(ii) Cubing both sides, we get

$\left(a+\frac{1}{a}\right)^3=(\pm3)^3$

$$\Rightarrow\ a^3+\frac{1}{a^3}+3\times a\times\frac{1}{a}\left(a+\frac{1}{a}\right)=\pm 27 \Rightarrow a^3+\frac{1}{a^3}+3(\pm 3)=\pm 27$$

$$\Rightarrow\ a^3+\frac{1}{a^3}\pm 9=\pm 27 \qquad \therefore\quad a^3+\frac{1}{a^3}=\pm 27-(\pm 9)=\pm 18$$

Q.13. If $\left(x^2+\frac{1}{25x^2}\right)=8\frac{3}{5}$, find the value of: (i) $\left(x+\frac{1}{5x}\right)$ (ii) $\left(x^3+\frac{1}{125x^3}\right)$

Ans. (i) $\left(x+\frac{1}{5x}\right)^2=x^2+\frac{1}{25x^2}+2\times x\times\frac{1}{5x}=x^2+\frac{1}{25x^2}+\frac{2}{5}=\frac{43}{5}+\frac{2}{5}=\frac{45}{5}=9$

$$\therefore\ x+\frac{1}{5x}=\pm\sqrt{9}=\pm 3$$

(ii) Cubing both sides, we get

$$\left(x+\frac{1}{5x}\right)^3=(\pm 3)^3$$

$$\Rightarrow\ (x^3)+\left(\frac{1}{5x}\right)^3+3\times x\times\frac{1}{5x}\left(x+\frac{1}{5x}\right)=\pm 27 \Rightarrow x^3+\frac{1}{125x^3}+\frac{3}{5}\times(\pm 3)=\pm 27$$

$$\Rightarrow\ x^3+\frac{1}{125x^3}\pm\frac{9}{5}=\pm 27 \Rightarrow x^3+\frac{1}{125x^3}=(\pm 27)-\left(\pm\frac{9}{5}\right)$$

$$\therefore\ x^3+\frac{1}{125x^3}=\pm\frac{126}{5}=\pm 25\frac{1}{5}$$

Q.14. If $\left(x+\frac{1}{x}\right)^2=3$, show that $\left(x^3+\frac{1}{x^3}\right)=0$.

Ans. $\left(x+\frac{1}{x}\right)^2=3 \ \therefore\ x+\frac{1}{x}=\pm\sqrt{3}$

Cubing both sides, we get

$$\left(x+\frac{1}{x}\right)^3=(\pm\sqrt{3})^3 \Rightarrow x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=\pm 3\sqrt{3}$$

$$\Rightarrow\ x^3+\frac{1}{x^3}+3\times(\pm\sqrt{3})=\pm 3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3}+(\pm 3\sqrt{3})=\pm 3\sqrt{3}$$

$$\Rightarrow\ x^3+\frac{1}{x^3}=\pm 3\sqrt{3}-(\pm 3\sqrt{3})=0$$

Hence, $x^3+\frac{1}{x^3}=0$

Q.15. If $\left(x-\frac{1}{x}\right)=4$, find the values of:

(i) $\left(x^2+\frac{1}{x^2}\right)$ **(ii)** $\left(x^4+\frac{1}{x^4}\right)$

Ans. (i) $x-\frac{1}{x}=4 \Rightarrow \left(x-\frac{1}{x}\right)^2=4^2$

Squaring both sides, we get

$\Rightarrow x^2+\frac{1}{x^2}-2=16 \Rightarrow x^2+\frac{1}{x^2}=16+2=18$

$\Rightarrow x^2+\frac{1}{x^2}=18$ ...(i)

(ii) On squaring both sides of eqn. (i), we get

$\left(x^2+\frac{1}{x^2}\right)^2=(18)^2 \Rightarrow x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}=324$

$\Rightarrow x^4+\frac{1}{x^4}+2=324 \Rightarrow x^4+\frac{1}{x^4}=324-2=322$

$\therefore x^4+\frac{1}{x^4}=322$

Q.16. If $a+\frac{1}{a}=p$; then show that :

$a^3+\frac{1}{a^3}=p(p^2-3)$

Ans. If $a+\frac{1}{a}=p$, show that, $a^3+\frac{1}{a^3}=p(p^2-3)$

$a+\frac{1}{a}=p$ (Given)

Cubing both sides, we get

$\therefore \left(a+\frac{1}{a}\right)^3=p^3$

$\Rightarrow a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)=p^3 \Rightarrow a^3+\frac{1}{a^3}=p^3-3(p)$

Hence, $a^3+\frac{1}{a^3}=p(p^2-3)$

Q.17. If $a+\frac{1}{a}=4$; find: (i) $a^2+\frac{1}{a^2}$ (ii) $a^4+\frac{1}{a^4}$

Ans. (i) If $a+\frac{1}{a}=4$

Squaring both sides, we get

$\Rightarrow \left(a+\frac{1}{a}\right)^2=(4)^2 \Rightarrow a^2+2a\times\frac{1}{a}+\frac{1}{a^2}=16$

$\Rightarrow a^2+2+\frac{1}{a^2}=16 \Rightarrow a^2+\frac{1}{a^2}=16-2 \Rightarrow a^2+\frac{1}{a^2}=14$

(ii) $a^4+\frac{1}{a^4}=\left(a^2+\frac{1}{a^2}\right)^2-2=(14)^2-2$ [since $a^2+\frac{1}{a^2}=14$ (from) (i)]

$=196-2=194$

Q.18. If $2x-3y=10$ and $xy=16$; find the value of $8x^3-27y^3$.

Ans. $2x-3y=10$ and $xy=16$

Cubing both sides, we get

$\therefore (2x-3y)^3=(10)^3$

$\Rightarrow 8x^3-27y^3-3(2x)(3y)(2x-3y)=1000$

$\Rightarrow 8x^3-27y^3-18xy(2x-3y)=1000$

$\Rightarrow 8x^3-27y^3-18(16)(10)=1000$

$\Rightarrow 8x^3-27y^3-2880=1000 \Rightarrow 8x^3-27y^3=3880$

Q.19. If $3a+5b+4c=0$, show that: $27a^3+125b^3+64c^3=180abc$.

Ans. $3a+5b+4c=0$

$\Rightarrow 3a+5b=-4c$

Cubing both sides, we get

$(3a+5b)^3=(-4c)^3$

$\Rightarrow (3a)^3+(5b)^3+3\times 3a\times 5b(3a+5b)=-64c^3$

$\Rightarrow 27a^3+125b^3+45ab\times(-4c)=-64c^3$

$\Rightarrow 27a^3+125b^3-180abc=-64c^3$

$\Rightarrow 27a^3+125b^3+64c^3=180abc$

Q.20. If $\frac{2x}{3y}=\frac{3y}{4z}$, show that : $4x^2+9y^2+16z^2=(2x-3y+4z)(2x+3y+4z)$

Ans. $\frac{2x}{3y}=\frac{3y}{4z}\Rightarrow 9y^2=8xz$ ...(i)

R.H.S. $=(2x-3y+4z)(2x+3y+4z)$

$=\{(2x+4z)-3y\}\{(2x+4z)+3y\}=(2x+4z)^2-(3y)^2$

$=4x^2+16xz+16z^2-9y^2=4x^2+2\times 8xz+16z^2-9y^2$

$=4x^2+2\times 9y^2+16z^2-9y^2$ (From eqn. (i) as $9y^2=8xz$)

$=4x^2+18y^2+16z^2-9y^2$

$=4x^2+9y^2+16z^2=$ L.H.S.

Q.21. If $(a+3b)=6$, show that $a^3+27b^3+54ab=216$.

Ans. $a+3b=6$

Cubing both sides, we get

$(a+3b)^3=(6)^3$

$\Rightarrow (a)^3+(3b)^3+3\times a\times 3b(a+3b)=216\Rightarrow a^3+27b^3+9ab(a+3b)=216$

$\Rightarrow a^3+27b^3+9ab\times 6=216\Rightarrow a^3+27b^3+54ab=216$

Q.22. If $a+2b+3c=0$, show that $a^3+8b^3+27c^3=18abc$.

Ans. $a+2b+3c=0$

$\Rightarrow a+2b=-3c$ (i)

Cubing both sides, we get

$(a+2b)^3=(-3c)^3$

$\Rightarrow (a)^3+(2b)^3+3\times a\times 2b(a+2b)=-27c^3$

$\Rightarrow a^3+8b^3+6ab\times(-3c)=-27c^3$ (From eqn. (i) as $a+2b=-3c$)

$\Rightarrow a^3+8b^3-18abc=-27c^3$

$\Rightarrow a^3+8b^3+27c^3=18abc$.

Q.23. Change the subject of each of the following to the letter given against them.

(i) $S=ut+\frac{1}{2}at^2$; a

(ii) $S=\frac{n}{2}[2a+(n-1)d]$; d

(iii) $i=\frac{nE}{nR+r}$; n

(iv) $F=\frac{9}{5}C+32$; C

(v) $n = \dfrac{1}{2\pi}\sqrt{\dfrac{8T}{lA}}$; A **(vi)** $S = u + \dfrac{1}{2}a(2t-1)$; t

Ans. (i) $S = ut + \dfrac{1}{2}at^2$ (Given) $\Rightarrow$ $s - ut = \dfrac{1}{2}at^2 \Rightarrow 2(s-ut) = at^2$

$$\Rightarrow at^2 = 2(s-ut) \Rightarrow a = \frac{2(s-ut)}{t^2}$$

(ii) $S = \dfrac{n}{2}[2a + (n-1)d]$ (Given)

$$\Rightarrow 2S = n[2a+(n-1)d] \Rightarrow \frac{2S}{n} = 2a + (n-1)d$$

$$\Rightarrow \frac{2S}{n} - 2a = (n-1)d \Rightarrow \frac{2S}{n} - \frac{2a}{1} = (n-1)d$$

$$\Rightarrow \frac{2S-2na}{n} = (n-1)d \Rightarrow \frac{2(S-na)}{n} = (n-1)d$$

$$\Rightarrow d = \frac{2(S-na)}{n(n-1)}$$

(iii) $I = \dfrac{nE}{nR+r}$ (Given)

$$\Rightarrow I \times (nR + r) = nE \Rightarrow InR + Ir = nE$$

$$\Rightarrow Ir = nE - InR \Rightarrow Ir = n(E - IR)$$

$$\Rightarrow n(E-IR) = Ir \Rightarrow n = \frac{Ir}{E-IR}$$

(iv) $F = \dfrac{9}{5}C + 32$ (Given)

$$\Rightarrow F - 32 = \frac{9}{5}C \Rightarrow 5(F-32) = 9C$$

$$\Rightarrow 9C = 5(F-32) \Rightarrow C = \frac{5}{9}(F-32)$$

(v) $n = \dfrac{1}{2\pi}\sqrt{\dfrac{8T}{lA}}$ (Given)

$$\Rightarrow 2\pi n = \sqrt{\frac{8T}{lA}}$$

Squaring both sides, we get

$$(2\pi n)^2 = \left(\sqrt{\frac{8T}{lA}}\right)^2 \Rightarrow 4\pi^2 n^2 = \frac{8T}{lA}$$

$\Rightarrow 4\pi^2 n^2 \times lA = 8T$ [By cross-multiplication, we get]

$$\Rightarrow A = \frac{8T}{4l\pi^2 n^2} = \frac{2T}{l\pi^2 n^2}$$

(vi) $S = u + \frac{1}{2}a\,(2t-1)$ (Given)

$$\Rightarrow s - u = \frac{1}{2}a(2t-1) \Rightarrow 2\,(s-u) = a\,(2t-1)$$

$$\Rightarrow a\,(2t-1) = 2\,(s-u) \Rightarrow 2t - 1 = \frac{2\,(s-u)}{a}$$

$$\Rightarrow 2t = \frac{2\,(s-u)}{a} + 1 \Rightarrow 2t = \frac{2\,(s-u)+a}{a} \Rightarrow t = \frac{2\,(s-u)+a}{2a}$$

Q.24. Given: $A = P\left(\mathbf{1} + \frac{rt}{\mathbf{100}}\right)$

(i) Express *t* in terms of *A*, *P* and *r*.

(ii) Find the value of *t* when $A = 605$, $P = 500$ and $r = 12$

Ans. We have: $A = P\left(1 + \frac{rt}{100}\right)$

$$\Rightarrow \frac{A}{P} = 1 + \frac{rt}{100}$$

$$\Rightarrow \frac{rt}{100} = \frac{A}{P} - 1$$

(i) $t = \frac{100}{r}\left(\frac{A}{P} - 1\right)$

(ii) When $A = 605$, $P = 500$ and $r = 12$, then

$$t = \frac{100}{12}\left(\frac{605}{500} - 1\right) = \frac{100}{12} \times \left(\frac{605-500}{500}\right)$$

$$\Rightarrow t = \frac{100}{12} \times \frac{105}{500} = \frac{7}{4} = 1\frac{3}{4}$$

Q.25. Given: $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$.

(i) In the above formula, make *u* the subject.

(ii) Find the value of *u* when $f=6$ and $v=10.5$.

Ans. (i) $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ (Given)

$$\Rightarrow \frac{1}{u}=\frac{1}{f}-\frac{1}{v}\Rightarrow \frac{1}{u}=\frac{v-f}{fv}$$

$$\Rightarrow u=\frac{fv}{v-f}$$

(ii) When $f=6,\ v=10.5$, then

$$u=\frac{6\times 10.5}{10.5-6}=\frac{63}{4.5}=\frac{63\times 10}{45}=14.$$

Q.26. (i) Make '*b*' the subject in the formula, $x=\sqrt{\frac{a-b}{a+b}}$

(ii) Find the value of *b* when $a=10$ and $x=\frac{1}{2}$.

Ans. (i) $x=\sqrt{\frac{a-b}{a+b}}$ (Given)

Squaring both sides, we get

$$x^2=\frac{a-b}{a+b}$$

$$\Rightarrow x^2a+x^2b=a-b\Rightarrow x^2b+b=a-ax^2$$

$$\Rightarrow b(x^2+1)=a(1-x^2)\Rightarrow b=\frac{a(1-x^2)}{1+x^2}$$

(ii) When $a=10$ and $x=\frac{1}{2}$, then

$$b=\frac{10\left[1-\left(\frac{1}{2}\right)^2\right]}{1+\left(\frac{1}{2}\right)^2}=\frac{10\left(1-\frac{1}{4}\right)}{1+\frac{1}{4}}=\frac{10\times\frac{3}{4}}{\frac{5}{4}}=10\times\frac{3}{4}\times\frac{4}{5}=6$$

Q.27. Make g the subject of formula:

$\frac{mv^2}{r} = T + mg$ **. Hence, find g, if $m = 1.2$, $v = 4$, $r = 2$ and $T = 2.4$.**

Ans. (i) $\frac{mv^2}{r} = T + mg$ (Given)

$$\Rightarrow \frac{mv^2}{r} - T = mg \Rightarrow mg = \frac{mv^2}{r} - T$$

$$\Rightarrow mg = \frac{mv^2 - Tr}{r} \Rightarrow g = \frac{mv^2 - Tr}{mr}$$

(ii) $g = \frac{mv^2 - Tr}{mr}$

Put, $m = 1.2$, $v = 4$, $r = 2$, $T = 2.4$

$$g = \frac{1.2 \times (4)^2 - 2.4 \times 2}{1.2 \times 2} \Rightarrow g = \frac{1.2 \times 16 - 2.4 \times 2}{2.4}$$

$$\Rightarrow g = \frac{19.2 - 4.8}{2.4} = \frac{14.4}{2.4} = \frac{144}{24} = 6$$

Q.28. The total energy E possessed by a body of mass 'm', moving with a velocity 'v' at a height 'h' is given by:

$$E = \frac{1}{2} mv^2 + mgh.$$

(i) Make 'm' the subject of formula.

(ii) Find m, if $v = 3$, $g = 10$, $h = 5$ and $E = 109$.

Ans. (i) $\text{E} = \frac{1}{2} mv^2 + mgh$ (Given)

$$\Rightarrow \text{E} = \frac{mv^2}{2} + \frac{mgh}{1} \Rightarrow \text{E} = \frac{mv^2 + 2mgh}{2}$$

$$\Rightarrow 2\text{E} = mv^2 + 2mgh \Rightarrow 2\text{E} = m\,(v^2 + 2gh)$$

$$\Rightarrow m\,(v^2 + 2gh) = 2\text{E}$$

$$\Rightarrow m = \frac{2\text{E}}{v^2 + 2gh}$$

(ii) $m = \dfrac{2E}{v^2 + 2gh}$ ...(i)

Where, $v = 3,\ g = 10,\ h = 5,\ E = 109,\ m = ?$

Putting the value of v, g, h and E in eqn. (i), we get

$$m = \frac{2 \times 109}{(3)^2 + 2 \times 10 \times 5} \Rightarrow m = \frac{218}{9 + 100} \Rightarrow m = \frac{218}{109} \Rightarrow m = 2$$

Q.29. (i) Make 'g' the subject of the formula, $T = 2\pi\sqrt{\dfrac{l}{g+k}}$.

(ii) Find g when $\pi = 3\frac{1}{7}$, $l = 539$, $T = 44$ and $k = 1.2$.

Ans. (i) $T = 2\pi\sqrt{\dfrac{l}{g+k}}$

$$\Rightarrow \frac{T}{2\pi} = \sqrt{\frac{l}{g+k}}$$

Squaring both sides, we get

$$\frac{T^2}{4\pi^2} = \frac{l}{g+k} \Rightarrow T^2 g + T^2 k = 4l\pi^2$$

$$\Rightarrow T^2 g = 4l\pi^2 - T^2 k \Rightarrow g = \frac{4l\pi^2 - T^2 k}{T^2}$$

(ii) When $\pi = 3\frac{1}{7} = \frac{22}{7}$, $l = 539$, $T = 44$ and $k = 1.2$

$$\therefore\quad g = \frac{4 \times 539 \times \left(\frac{22}{7}\right)^2 - (44)^2 \times 1.2}{(44)^2}$$

$$= \frac{\frac{4 \times 539 \times 484}{49} - 1936 \times 1.2}{1936} = \frac{21296 - 2323.2}{1936} = \frac{18972.8}{1936} = 9.8$$

Q.30. Factorise:

(i) $ab\,(x^2+y^2)-xy\,(a^2+b^2)$ **(ii)** $a^3+ab\,(\mathbf{1}-\mathbf{2}a)-\mathbf{2}b^2$

(iii) $x^2+\dfrac{\mathbf{1}}{x^2}-\mathbf{2}-\mathbf{3}x+\dfrac{\mathbf{3}}{x}$ **(iv)** $\mathbf{9}x^4-x^2-\mathbf{12}x-\mathbf{36}$

(v) $x^2+\dfrac{\mathbf{1}}{x^2}-\mathbf{11}$ **(vi)** $a^2+b^2-c^2-d^2+\mathbf{2}ab-\mathbf{2}cd$

Ans. (i) $ab\,(x^2+y^2)-xy\,(a^2+b^2)=abx^2+aby^2-xya^2-xyb^2$

$$=abx^2-xya^2-xyb^2+aby^2$$
$$=ax\,(bx-ay)-by\,(bx-ay)$$
$$=(bx-ay)\,(ax-by)$$

(ii) $a^3+ab\,(1-2a)-2b^2=a^3+ab-2a^2b-2b^2=a\,(a^2+b)-2b\,(a^2+b)$

$$=(a^2+b)\,(a-2b)$$

(iii) $x^2+\dfrac{1}{x^2}-2-3x+\dfrac{3}{x}=\left(x-\dfrac{1}{x}\right)^2-3\left(x-\dfrac{1}{x}\right)$

$$=\left(x-\frac{1}{x}\right)\left(x-\frac{1}{x}-3\right)$$

(iv) $9x^4-x^2-12x-36=9x^4-(x^2+12x+36)$

$$=(3x^2)^2-\{(x)^2+2\times x\times 6+(6)^2\}=(3x^2)^2-(x+6)^2$$
$$=(3x^2+x+6)\,(3x^2-x-6)$$

(v) $x^2+\dfrac{1}{x^2}-11=\left(x^2+\dfrac{1}{x^2}-2\right)-9$

$$=\left(x-\frac{1}{x}\right)^2-(3)^2=\left(x-\frac{1}{x}+3\right)\left(x-\frac{1}{x}-3\right)$$

(vi) $a^2+b^2-c^2-d^2+2ab-2cd=a^2+b^2+2ab-c^2-d^2-2cd$

$$=(a^2+b^2+2ab)-(c^2+d^2+2cd)$$
$$=(a+b)^2-(c+d)^2$$
$$=(a+b+c+d)\,(a+b-c-d).$$

Q.31. Factorise:

(i) $2x^2 - 7x - 39$ **(ii)** $10 + 3x - x^2$

(iii) $x^2 - 3ax - 88a^2$ **(iv)** $24x^3 + 37x^2 - 5x$

(v) $x(2x - y) - y^2$ **(vi)** $5x^2 + 17xy - 12y^2$

Ans. (i) $2x^2 - 7x - 39 = 2x^2 - 13x + 6x - 39 = x(2x - 13) + 3(2x - 13)$

$= (2x - 13)(x + 3)$

(ii) $10 + 3x - x^2 = 10 + 5x - 2x - x^2 = 5(2 + x) - x(2 + x)$

$= (2 + x)(5 - x)$

(iii) $x^2 - 3ax - 88a^2 = x^2 - 11ax + 8ax - 88a^2 = x(x - 11a) + 8a(x - 11a)$

$= (x - 11a)(x + 8a)$

(iv) $24x^3 + 37x^2 - 5x = x(24x^2 + 37x - 5) = x(24x^2 + 40x - 3x - 5)$

$= x\{8x(3x + 5) - 1(3x + 5)\}$

$= x(3x + 5)(8x - 1)$

(v) $x(2x - y) - y^2 = 2x^2 - xy - y^2 = 2x^2 - 2xy + xy - y^2$

$= 2x(x - y) + y(x - y) = (x - y)(2x + y)$

(vi) $5x^2 + 17xy - 12y^2 = 5x^2 + 20xy - 3xy - 12y^2 = 5x(x + 4y) - 3y(x + 4y)$

$= (x + 4y)(5x - 3y)$

Q.32. Factorise:

(i) $5(3a + b)^2 + 6(3a + b) - 8$ **(ii)** $3(2a - b)^2 - 19(2a - b) + 28$

(iii) $(3x - 2y)^2 + 3(3x - 2y) + 10$ **(iv)** $(a^2 - 3a)(a^2 - 3a + 7) + 10$

Ans. (i) $5(3a + b)^2 + 6(3a + b) - 8$

Let $3a + b = x$, then

$5(3a + b)^2 + 6(3a + b) - 8 = 5x^2 + 6x - 8$

$= 5x^2 + 10x - 4x - 8 = 5x(x + 2) - 4(x + 2)$

$= (x + 2)(5x - 4)$

Putting the value of x, we get

$5(3a + b)^2 + 6(3a + b) - 8 = (3a + b + 2)\{5(3a + b) - 4\}$

$= (3a + b + 2)(15a + 5b - 4)$

(ii) Let $2a-b=x$, then

$$3(2a-b)^2-19(2a-b)+28=3x^2-19x+28=3x^2-12x-7x+28$$
$$=3x(x-4)-7(x-4)=(x-4)(3x-7)$$

Putting the value of x, we get

$$3(2a-b)^2-19(2a-b)+28=(2a-b-4)\{3(2a-b)-7\}$$
$$=(2a-b-4)(6a-3b-7)$$

(iii) $(3x-2y)^2+3(3x-2y)-10=a^2+3a-10=a^2+5a-2a-10$

$$=a(a+5)-2(a+5)=(a+5)(a-2)$$

Putting $a=3x-2y$, we get :

$$(3x-2y)^2+3(3x-2y)-10=(3x-2y+5)(3x-2y-2)$$

(iv) $(a^2-3a)(a^2-3a+7)+10=(a^2-3a)(a^2-3a)+7(a^2-3a)+10$

$$=(a^2-3a)^2+7(a^2-3a)+10$$

Let $a^2-3a=x$, then

$$(a^2-3a)^2+7(a^2-3a)+10=x^2+7x+10=x^2+5x+2x+10$$
$$=x(x+5)+2(x+5)=(x+5)(x+2)$$

Putting the value of x, we get

$$=(a^2-3a+5)(a^2-3a+2)=(a^2-3a+5)\{a^2-2a-a+2\}$$
$$=(a^2-3a+5)\{a(a-2)-1(a-2)\}=(a^2-3a+5)(a-2)(a-1)$$

Q.33. Factorise:

(i) $x^2 + \frac{1}{4x^2} + 1 - 7x - \frac{7}{2x}$ **(ii)** $9a^2 + \frac{1}{9a^2} - 2 - 12a + \frac{4}{3a}$

(iii) $x^2 + \frac{a^2+1}{a}x + 1$ **(iv)** $x^2 + \frac{1}{x^2} - 3$

Ans. (i) $x^2 + \frac{1}{4x^2} + 1 - 7x - \frac{7}{2x}$

$$= (x)^2 + \left(\frac{1}{2x}\right)^2 + 2 \times x \times \frac{1}{2x} - 7\left(x + \frac{1}{2x}\right)$$

$$= \left(x + \frac{1}{2x}\right)^2 - 7\left(x + \frac{1}{2x}\right)$$

$$= \left(x + \frac{1}{2x}\right)\left(x + \frac{1}{2x} - 7\right)$$

$$= \left(x + \frac{1}{2x}\right)\left(x - 7 + \frac{1}{2x}\right)$$

(ii) $9a^2 + \frac{1}{9a^2} - 2 - 12a + \frac{4}{3a} = (3a)^2 + \left(\frac{1}{3a}\right)^2 - 2 \times 3a \times \frac{1}{3a} - 4\left(3a - \frac{1}{3a}\right)$

$$= \left(3a - \frac{1}{3a}\right)^2 - 4\left(3a - \frac{1}{3a}\right) = \left(3a - \frac{1}{3a}\right)\left(3a - \frac{1}{3a} - 4\right)$$

(iii) $x^2 + \frac{a^2+1}{a}x + 1 = x^2 + ax + \frac{1}{a}x + 1$

$$= x\,(x + a) + \frac{1}{a}(x + a) = (x + a)\left(x + \frac{1}{a}\right)$$

(iv) $x^2 + \frac{1}{x^2} - 3 = x^2 + \frac{1}{x^2} - 2 - 1$

$$= \left(x - \frac{1}{x}\right)^2 - (1)^2 \qquad \text{[Using } a^2 - b^2 = (a+b)\,(a-b)\text{]}$$

$$= \left(x - \frac{1}{x} + 1\right)\left(x - \frac{1}{x} - 1\right) = \left(x + 1 - \frac{1}{x}\right)\left(x - 1 - \frac{1}{x}\right)$$