Question Bank
Quadratic Equations

1. Solve the following equations for x :

(i) $8x^2 + 15 = 26x$ (ii) $x(2x + 5) = 25$

Solution.

(i) Given $8x^2 + 15 = 26x$

$\Rightarrow$ $8x^2 - 26x + 15 = 0$

[Putting in the form as $ax^2 + bx + c = 0$]

$\Rightarrow$ $8x^2 - 20x - 6x + 15 = 0$

$\Rightarrow$ $4x(2x - 5) - 3(2x - 5) = 0$

$\Rightarrow$ $(2x - 5)(4x - 3) = 0$

$\Rightarrow$ $2x - 5 = 0$ or $4x - 3 = 0$ [Zero product rule]

$\Rightarrow$ $x = \frac{5}{2}$ or $x = \frac{3}{4}$

Hence, the roots of the given equation are $\frac{5}{2}, \frac{3}{4}$.

(ii) Given $x(2x + 5) = 25$

$\Rightarrow$ $2x^2 + 5x - 25 = 0$

[Putting in the form as $ax^2 + bx + c = 0$]

$\Rightarrow$ $2x^2 + 10x - 5x - 25 = 0$

$\Rightarrow$ $2x(x + 5) - 5(x + 5) = 0$

$\Rightarrow$ $(x + 5)(2x - 5) = 0$

$\Rightarrow$ $x + 5 = 0$ or $2x - 5 = 0$

$\Rightarrow$ $x = -5$ or $x = \frac{5}{2}$

Hence, the roots of the given equation are $-5, \frac{5}{2}$.

2. Solve $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$, $x \neq -3$, $x \neq 3$

Solution. We have, $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2} = \frac{5}{2}$

Multiplying both sides by $2(x + 3)(x - 3)$, L.C.M. of fractions, we get

$$2(x-3)^2 + 2(x+3)^2 = 5(x+3)(x-3)$$

$\Rightarrow$ $2(x^2 - 6x + 9) + 2(x^2 + 6I + 9) = 5(x^2 - 9)$

$\Rightarrow$ $2x^2 - 12x + 18 + 2x^2 + 12x + 18 = 5x^2 - 45$

$\Rightarrow$ $4x^2 + 36 = 5x^2 - 45$

$\Rightarrow$ $4x^2 - 5x^2 + 36 + 45 = 0$

$\Rightarrow$ $81 - x^2 = 0$

$\Rightarrow$ $x^2 - 81 = 0$ [Putting in the form $ax^2 + bx + c = 0$]

$\Rightarrow$ $(x - 9)(x + 9) = 0$

$\Rightarrow$ $x - 9 = 0$ or $x + 9 = 0$ [Zero-product rule]

$\Rightarrow$ $x = 9$ or $x = -9$

Hence, the roots of the given equation are 9, – 9.

3. Solve $2x - 3 = \sqrt{2x^2 - 2x + 21}$

Solution. Given, $2x - 3 = \sqrt{2x^2 - 2x + 21}$

On squaring both sides, we get

$(2x - 3)2 = 2x^2 - 2x + 21$

$\Rightarrow$ $4x^2 - 12x + 9 = 2x^2 - 2x + 21$

$\Rightarrow$ $4x^2 - 2x^2 - 12x + 2x + 9 - 21 = 0$

$\Rightarrow$ $2x^2 - 10x - 12 = 0$

$\Rightarrow$ $x^2 - 5x - 6 = 0$

$\Rightarrow$ $(x - 6)(x + 1) = 0$

$\Rightarrow$ $x - 6 = 0$ or $x + 1 = 0$ [Zero product rule]

$\Rightarrow$ $x = 6$ or $x = -1$

But, $x = -1$ does not satisfy the given equation, so -1 is rejected.

Hence, the solution is 6.

4. Solve $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{(a+b+x)}$

Solution. We have $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$

$\Rightarrow \quad \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{x} + \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \quad \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow \quad x(a+b+x) = -ab$

$\Rightarrow \quad x^2 + (a+b)x + ab = 0$

$\Rightarrow \quad x^2 + ax + bx + ab = 0$

$\Rightarrow \quad x(x+a) + b(x+a) = 0$

$\Rightarrow \quad (x+a)(x+b) = 0$

$\Rightarrow \quad x = -a$ or $x = -b$

Hence, the roots of the given equation are $-a, -b$

5. Solve : $2^{2x} - 3 \times 2^{x+2} + 32 = 0$

Solution. We have $2^{2x} - 3 \times 2^{x+2} + 32 = 0$

$\Rightarrow \quad 2^{2x} - 3 \times 2^x \times 2^2 + 32 = 0$

$\Rightarrow \quad 2^{2x} - 12 \times 2^x + 32 = 0$

$\Rightarrow \quad y^2 - 12y + 32 = 0$, where $y = 2^x$

$\Rightarrow \quad y^2 - 8y - 4y + 32 = 0$

$\Rightarrow \quad y(y-8) - 4(y-8) = 0$

$\Rightarrow \quad (y-8)(y-4) = 0$

$\Rightarrow \quad y = 8$ or $y = 4$

Now, $y = 8 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$

And $y = 4 \Rightarrow 2^x = 4 \Rightarrow 2^x = 2^2 \Rightarrow x = 2$

Hence, roots of the given equation are 2, 3

6. Solve : $\sqrt{4-x} + \sqrt{x+9} = 5$

Solution. We have $\sqrt{4-x} + \sqrt{x+9} = 5$

$\Rightarrow \sqrt{4-x} = 5 - \sqrt{x+9}$

Squaring both sides, we get

$4 - x = 25 + (x + 9) - 10\sqrt{x+9}$

$\Rightarrow 4 - x - 34 - x = -10\sqrt{x+9}$

$\Rightarrow 10\sqrt{(x+9)} = 2(x + 15)$

$\Rightarrow 5\sqrt{x+9} = x + 15$

Again squaring both sides, we get

$25(x + 9) = x^2 + 225 + 30x$

$\Rightarrow x^2 + 30x + 225 - 25x - 225 = 0$

$\Rightarrow x^2 + 5x = 0$

$\Rightarrow x(x + 5) = 0$

$\Rightarrow x = 0$ or $x = -5$

Hence, roots of the given equation are 0, –5

7. Solve the equation $2x - \frac{1}{x} = 7$. Write your answers correct to two decimal places.

Solution. We have $2x - \frac{1}{x} = 7 \Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$ …(i)

Comparing (i) with $ax^2 + bx + c$, we get, $a = 2, b = -7, c = -1$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7 \pm \sqrt{49 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{49 + 8}}{4} = \frac{7 \pm \sqrt{57}}{4}$$

$\Rightarrow \quad x = \dfrac{7 + \sqrt{57}}{4}$ or $x = \dfrac{7 - \sqrt{57}}{4}$

$\Rightarrow \quad x = \dfrac{7 + 7.55}{4}$ or $x = \dfrac{7 - 7.55}{4}$

$\Rightarrow \quad x = \dfrac{14.55}{4}$ or $x = -\dfrac{0.55}{4}$

$\Rightarrow \quad x = 3.64 \quad$ or $\quad x = -0.14$

8. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places.

Solution. We have $3x^2 - x - 7 = 0$ …(i)

Comparing (i) with $ax^2 + bx + c = 0$, we get

$a = 3, \qquad b = -1, \qquad c = -7$

$\therefore$ By quadratic formula, we have

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$= \frac{1 \pm \sqrt{1 + 84}}{6} = \frac{1 \pm \sqrt{85}}{6}$$

$$= \frac{1 \pm 9.22}{6} = 1.70 \text{ or } -1.37$$

9. Solve : $\dfrac{1}{(x + 1)} + \dfrac{2}{(x + 2)} = \dfrac{4}{(x + 4)}$

Solution.

Given $\dfrac{1}{(x + 1)} + \dfrac{2}{(x + 2)} = \dfrac{4}{(x + 4)}$

$$\Rightarrow \quad \frac{(x + 2) + 2(x + 1)}{(x + 1)(x + 2)} = \frac{4}{(x + 4)}$$

$$\Rightarrow \quad \frac{(3x + 4)}{(x + 1)(x + 2)} = \frac{4}{(x + 4)}$$

$\Rightarrow \quad (3x + 4)(x + 4) = 4(x + 1)(x + 2)$

$\Rightarrow \quad 3x^2 + 16x + 16 = 4x^2 + 12x + 8$

$\Rightarrow \quad x^2 - 4x - 8 = 0$ . ..(i)

Comparing (i) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -4$ and $c = -8$

$\therefore$ By quadratic formula, we have :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 1, b = -4 \text{ and } c = -8$$

$$\Rightarrow \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1(-8)}}{2 \times 1}$$

$$\Rightarrow \quad x = \frac{4 \pm \sqrt{16 + 32}}{2} \Rightarrow x = \frac{4 \pm \sqrt{48}}{2}$$

$$\Rightarrow \quad x = \frac{4 \pm 4\sqrt{3}}{2} \quad \Rightarrow x = 2 \pm 2\sqrt{3}$$

$$\Rightarrow \quad x = \left(2 + 2\sqrt{3}\right) \text{ or } x = \left(2 - 2\sqrt{3}\right)$$

10. Solve : $(x + 1)(x + 2)(x + 3)(x + 4) = 24$

Solution. We have $(x + 1)(x + 4)(x + 2)(x + 3) = 24$

$\Rightarrow \quad (x^2 + 5x + 4)(x^2 + 5x + 6) = 24$

Let $x^2 + 5x = y$. Then

$(y + 4)(y + 6) - 24 = 0 \Rightarrow y^2 + 10y + 24 - 24 = 0$

$\Rightarrow \quad y^2 + 10y = 0 \Rightarrow y(y + 10) = 0$

$\Rightarrow \quad y = 0$ or $y = -10$

Now $y = 0 \Rightarrow x^2 + 5x = 0$

$\Rightarrow \quad x(x + 5) = 0$

$\Rightarrow \quad x = 0$ or $x = -5$

And $y = -10 \Rightarrow x^2 + 5x = -10$

$\Rightarrow \quad x^2 + 5x + 10 = 0$

Comparing the above equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 5$, $c = 10$

$$\therefore \quad x = \frac{5 \pm \sqrt{25 - 4 \times 1 \times 10}}{2 \times 1} = \frac{5 \pm \sqrt{-15}}{2}, \text{ which is not real}$$

Hence, $x = 0, -5$

11. Solve : $9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

Solution. We have $9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$ …(i)

Let $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore$ (i) becomes $9(y^2 - 2) - 9y - 52 = 0$

$\Rightarrow \quad 9y^2 - 18 - 9y - 52 = 0$

$\Rightarrow \quad 9y^2 - 9y - 70 = 0$

$\Rightarrow \quad 9y^2 - 30y + 21y - 70 = 0$

$\Rightarrow \quad 3y(3y - 10) + 7(3y - 10) = 0$

$\Rightarrow \quad (3y - 10)(3y + 7) = 0$

$\Rightarrow \quad y = \frac{10}{3} \quad$ or $\quad = -\frac{7}{3}$

Now, $\quad y = \frac{10}{3} \Rightarrow x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{10}{3}$

$\Rightarrow \quad 3x^2 + 3 = 10x$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x - 3) - 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(3x - 1) = 0$

$\Rightarrow \quad x = 3$ or $x = \frac{1}{3}$

And $y = \frac{-7}{3} \Rightarrow x + \frac{1}{x} = \frac{-7}{3}$

$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{-7}{3} \Rightarrow 3x^2 + 3 = -7x$

$\Rightarrow \quad 3x^2 + 7x + 3 = 0$

$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times 3}}{2 \times 3} = \frac{-7 \pm \sqrt{13}}{6}$$

Hence, $x = 3, \frac{1}{3} \quad \frac{-7 \pm \sqrt{13}}{6}$

12. Comment upon the nature of roots of each of the following equations, without solving it :

(i) $4x^2 - 20x + 25 = 0$ (ii) $3x^2 + 8x + 2 = 0$

(iii) $2x^2 - 9x + 7 = 0$ (iv) $11x^2 - 4\sqrt{5^x} + 1 = 0$

(v) $7x^2 + 5x + 4 = 0$

Solution.

(i) Here, $a = 4$, $b = -20$, $c = 25$

$\therefore$ $D = b^2 - 4ac = 400 - 4 \times 4 \times 25$

$400 - 400 = 0$

$\because$ D = 0, the roots are real and equal, each equal to

$\frac{20}{2 \times 4}$ i.e., $\frac{5}{2}$

(ii) Here, $a = 3$, $b = 8$, $c = 2$

$\therefore$ $D = b^2 - 4ac = 64 - 4 \times 3 \times 2 = 64 - 24 = 40$

$\because$ D > 0 and not a perfect square, thus, the roots are irrational and unequal.

(iii) Here, $a = 2$, $b = -9$, $c = 7$

$\therefore$ $D = b^2 - 4ac = 81 - 4 \times 2 \times 7 = 81 - 56 = 25$

$\because$ D > 0 and a perfect square, thus, the roots are unequal and rational.

(iv) Here, $a = 11$, $b = -4\sqrt{5}$, $c = 1$.

$\therefore$ $D = b^2 - 4ac = 80 - 4 \times 11 \times 1 = 80 - 44 = 36$

$\because$ D > 0 and a perfect square, but b is irrational, hence, the roots are unequal and irrational

(v) Hence, $a = 7$, $b = 5$, $c = 4$

$\therefore$ $D = b^2 - 4ac = 25 - 4 \times 7 \times 4 = 25 - 112 = -87$

$\because$ D < 0, therefore, the roots are unequal and imaginary.

13. Show that both the roots of $(x - p)(x - q) = a^2$ are always real.

Solution. We have, $(x - p)(x - q) = a^2$

$\Rightarrow \quad x - (p + q)x + pq = a^2$

$\Rightarrow \quad x^2 - (p + q)x + pq - a^2 = 0$

Here, $a = 1$, $b = -(p + q)$, $c = pq - a^2$

$\therefore \quad D = b^2 - 4ac = (p + q)^2 - 4 \times 1 \times (pq - a^2)$

$= p^2 + q^2 + 2pq - 4pq + 4a^2$

$= (p - q)^2 + 4a^2$

$\Rightarrow$ D is always > 0 as sum of the squares of two numbers can never benegative. Hence, the roots are always real.

14. Find the values of k for which :

(i) $x^2 - 2kx + 7k = 0$ has equal roots.

(ii) $(3k + 1)x^2 + 2(k + 1)x + k = 0$ has equal roots.

(iii) $5x^2 - kx + 1 = 0$ has real and distinct roots.

(iv) $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.

Solution.

(i) Here, $a = 1$, $b = -2k$, $c = 7$

For equal roots D = 0

$\Rightarrow \; b^2 - 4ac = 0 \;\Rightarrow\; 4k^2 - 4 \times 1 \times 7 = 0$

$\Rightarrow \; 4k^2 = 28 \;\Rightarrow\; k^2 = 7 \Rightarrow k = \pm\sqrt{7}$

(ii) Here, $a = (3k + 1)$, $b = 2(k + 1)$, $c = k$

For equal roots D = 0

$\Rightarrow \; b^2 - 4ac = 0$

$\Rightarrow \; \{2(k + 1)\}^2 - 4 \times (3k + 1) \times k = 0$

$\Rightarrow \; 4k^2 + 4 + 8k - 12k^2 - 4k = 0$

$\Rightarrow \; -8k^2 + 4k + 4 = 0$

$\Rightarrow \; 2k^2 - k - 1 = 0$

$$\Rightarrow \; k = \frac{1 \pm \sqrt{1 + 4 \times 2 \times 1}}{2 \times 2} = \frac{1 \pm 3}{4}$$

$$\Rightarrow \quad k = \frac{1 + 3}{4} \text{ or } k = \frac{1 - 3}{4}$$

$\Rightarrow \; k = 1 \text{ or } k = -\frac{1}{2}$

(iii) Here, $a = 5$, $b = -k$, $c = 1$

For real and distinct roots D > 0

$\Rightarrow \quad b^2 - 4ac > 0$

$\Rightarrow \quad k^2 - 4 \times 5 \times 1 > 0 \Rightarrow k^2 > 20$

$\Rightarrow \; k > \sqrt{20} \text{ or } k < -\sqrt{20}$

(iv) Here, $a = (4 - k)$, $b = (2k + 4)$, $c = 8k + 1$

The given equation will be a perfect square, if its discriminant is 0. i.e., D = 0.

$\Rightarrow \; b^2 - 4ac = 0$

$\Rightarrow \; (2k + 4)^2 - 4 \times (4 - k)(8k + 1) = 0$

$\Rightarrow \; 4k^2 + 16 + 16k - 4(32k + 4 - 8k^2 - k) = 0$

$\Rightarrow \; 4k^2 + 16 + 16k - 124k - 16 + 32k^2 = 0$

$\Rightarrow \; 36k^2 - 108k = 0$

$\Rightarrow \; k(36k - 108) = 0$

$\Rightarrow \; k = 0 \text{ or } k = \frac{108}{36} = 3$

15. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are real and equal, show that either $a^3 + b^3 + c^3 = 3abc$ or $a = 0$.

Solution.

Here, $A = c^2 - ab$, $B = -2(a^2 - bc)$, $C = b^2 - ac$

$$\begin{aligned}
\Rightarrow \; D &= B^2 - 4AC \\
&= 4(a^2 - bc)^2 - 4 \times (c^2 - ab)(b^2 - ac) \\
&= 4a^4 + 4b^2c^2 - 8a^2bc - 4(c^2b^2 - c^3a - ab^3 + a^2bc) \\
&= 4a^4 + 4b^2c^2 - 8a^2bc - 4b^2c^2 + 4ac^3 + 4ab^3 - 4a^2bc \\
&= 4a^4 - 12a^2bc + 4ac^3 + 4ab^3 \\
&= 4a(a^3 + b^3 + c^3 - 3abc)
\end{aligned}$$

For real and equal roots D = 0

$\therefore$ either $a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow$ either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

16. Show that the roots of the equation $(a + b + c)x^2 - 2(a + b)x + (a + b - c) = 0$ are rational.

Solution.

Here, A = $a + b + c$, B = $-2(a + b)$, C = $(a + b - c)$

$\therefore$ D $= B^2 - 4AC$

$= 4(a^2 + b^2 + 2ab) - 4(a + b + c)(a + b - c)$

$= 4a^2 + 4b^2 + 8ab - 4\{(a + b)^2 - c^2\}$

$= 4a^2 + 4b^2 + 8ab - 4(a^2 + b^2 + 2ab - c^2)$

$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 - 8ab + 4c^2$

$= 4c^2 = (2c)^2$

$\because$ D ≥ 0 and *a* perfect square hence the roots are rational.

17. Determine the positive value of m for which the equations $x^2 + mx + 64 = 0$ and $x^2 - 8x + m = 0$ will both have real roots.

Solution.

For $x^2 + mx + 64 = 0$, $a = 1$, $b = m$, $c = 64$

For real roots D ≥ 0.

$\Rightarrow b^2 - 4ac \geq 0 \Rightarrow m^2 - 4 \times 1 \times 64 > 0$

$\Rightarrow m^2 \geq 4 \times 64 \Rightarrow m > 16$ ...(i) [$\because$ Q m is + ve]

For $x^2 - 8x + m = 0$, $a = 1$, b = -8, $c = m$

For real roots, D ≥ 0

$\Rightarrow b^2 - 4ac > 0 \Rightarrow 64 - 4 \times 1 \times m \geq 0$

$\Rightarrow 64 \geq 4m \Rightarrow m \leq 16$ ...(ii)

From (i) and (ii), we have $m = 16$

18. If the roots of the equation $2x^2 - 2cx + ab = 0$ be real and distinct, show that the roots of $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ will be imaginary.

Solution.

For $2x^2 - 2cx + ab = 0$,

D $= 4c^2 - 4 \times 2 \times ab = 4c^2 - 8ab$

For real and distinct roots D > 0

$\Rightarrow 4c^2 - 8ab \geq 0 \Rightarrow 4c^2 \geq 8ab$ ...(i)

Now, for $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$,

D $= 4(a^2 + b^2 + 2ab) - 4 \times 1 \times (a^2 + b^2 + c^2)$

$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 - 4c^2$

$= 8ab - 4c^2 \leq 0$, since from (i), $4c^2 \geq 8ab$.

Hence, roots of the equation

$x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ are imaginary

19. An aeroplane travelled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for :

(i) the onward journey (ii) the return journey

If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x.

Solution. (i) Speed of the aeroplane on onward journey

$= x$ km/hr (Given)

Distance covered $= 400$ km

$\therefore$ Time taken for onward journey $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \left[\dfrac{400}{x}\right]$ hrs.

(ii) Speed of the aeroplane on the return journey

$= (x + 40)$ km/hr

Distance covered $= 400$ km

$\therefore$ Time taken for the return journey $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{400}{(x+40)}$ hrs.

$$\therefore \quad \frac{400}{x} - \frac{400}{(x+40)} = \frac{30}{60}$$

$$\Rightarrow \quad = \frac{1}{x} - \frac{1}{(x+40)} = \frac{1}{(2 \times 400)}$$

$$\Rightarrow \quad = \frac{(x+40) - x}{x\,(x+40)} = \frac{1}{800}$$

$$\Rightarrow \quad x\,(x + 40) = 32000$$

$$\Rightarrow \quad x^2 + 40x - 32000 = 0$$

$$\Rightarrow \quad x^2 + 200x - 160x - 32000 = 0$$

$\Rightarrow$ $x(x + 200) - 160(x + 200) = 0$

$\Rightarrow$ $(x + 200)(x - 160) = 0$

$\Rightarrow$ $x = -200$ or $x = 160$

$\Rightarrow$ $x = 160$

[Rejecting the negative value as speed cannot be negative.]

Hence, $x = 160$ km/hr.

20. Car A travels x km for every litres of petrol, while car B travels $(x + 5)$ km for every litre of petrol.

(i) Write down the number of litres of petrol used by car A and car B in covering a distance of 400 km.

(ii) If car A uses 4 litres of petrol more than car B in covering the 400 km, write down an equation in terms of x and solve it to determine the number of litres of petrol used by car B for the journey.

Solution. Distance = 400 km, Car A travels x km/litre, Car B travels $(x + 5)$ km/litre

(i) No. of litres of petrol used by car A = $\frac{400}{x}$

No. of litres of petrol used by car B = $\frac{400}{x+5}$

(ii) Car A uses 4 litres of petrol more than car B

$\therefore$ $\frac{400}{x} - \frac{400}{x+5} = 4$

$\Rightarrow$ $400(x + 5) - 400x = 4x(x + 5)$

$\Rightarrow$ $400x + 2000 - 400x = 4x^2 + 20x$

$\Rightarrow$ $4x^2 + 20x - 2000 = 0$

$\Rightarrow$ $x^2 + 5x - 500 = 0$

$\Rightarrow$ $x^2 + 25x - 20x - 500 = 0$

$\Rightarrow$ $x(x + 25) - 20(x + 25) = 0$

$\Rightarrow$ $(x + 25)(x - 20) = 0$

$\Rightarrow$ $x + 25 = 0$ or $x - 20 = 0$

$\Rightarrow$ $x = -25$ or 20

$\therefore$ $x = 20$ (Rejecting the negative value)

$\therefore$ Car A travels 20 km for every litre.
and Car B travels 25 km for every litre.

$\therefore$ No. of litres of petrol used by car B $= \dfrac{400}{25} = 16$ litres

21. A number of students decided to go on a picnic and planned to spend Rs 96 on eatables. Four of them however did not turn up. As a consequence, the remaining ones had to contribute Rs 2 each extra. Find the number of those who attended the picnic.

Solution. Let x students attended the picnic.

Then expense on eatables per head = Rs $\dfrac{96}{x}$

No. of students who planned the picnic = $x + 4$

Then in this case, expense on eatables per head = Rs $\dfrac{96}{x+4}$

But the difference in the expenses = Rs 2

$$\therefore \quad \frac{96}{x} - \frac{96}{x+4} = 2 \Rightarrow \frac{1}{x} - \frac{1}{x+4} = \frac{1}{48}$$

$$\Rightarrow \quad \frac{4}{x^2 + 4x} = \frac{1}{48} \Rightarrow x^2 + 4x - 192 = 0$$

$\Rightarrow \quad x^2 + 16x - 12x - 192 = 0$

$\Rightarrow \quad x(x + 16) - 12(x + 16) = 0$

$\Rightarrow \quad (x + 16)(x - 12) = 0$

$\Rightarrow \quad x = -16$ or $x = 12$

$\Rightarrow \quad x = 12$ [Rejecting $x = -16$, since number of students cannot be negative]

Hence, 12 students attended the picnic

22. Two pipes working together can fill a tank in 35 minutes. If the larger pipe alone can fill the tank in 24 minutes less than the time taken by the smaller pipe, then find the time taken by each pipe working alone to fill the tank.

Solution. Let the larger pipe take x minutes to fill the tank. Then The smaller pipe will take $(x + 24)$ minutes to fill the tank.

Part of the tank filled by larger pipe in 1 minute = $\frac{1}{x}$

Part of the tank filled by smaller pipe in 1 minute = $\frac{1}{x+24}$

Part of the tank filled by both the pipes in 1 minute

$$= \frac{1}{x} + \frac{1}{x+24} = \quad \text{... (i)}$$

Also, both the pipes together fill the tank in 35 minutes.

Part of the tank filled by both the pipes in 1 minute = $\frac{1}{35}$ ... (ii)

From (i) and (ii), $\frac{1}{x} + \frac{1}{x+24} = \frac{1}{35}$

$$\frac{2x+24}{x^2+24x} = \frac{1}{35}$$

$\Rightarrow \quad \Rightarrow 70x + 840 = x^2 + 24x$

$\Rightarrow x^2 - 46x - 840 = 0 \Rightarrow x^2 - 60x + 14x - 840 = 0$

$\Rightarrow x(x - 60) + 14(x - 60) = 0$

$\Rightarrow (x - 60)(x + 14) = 0$

$\Rightarrow x = 60$ or $x = -14$

$\Rightarrow x = 60$ [Rejecting $x = -14$, since time cannot be negative]

Hence, larger pipe can fill the tank in 60 minutes and the smaller pipe can fill the tank in 60 + 24 = 84 minutes

23. A farmer prepares a rectangular vegetable garden of area 180 m^2. With 39 m of barbed wire, we can fence the three sides of the garden, leaving one of the longer side unfenced. Find the dimension of the garden.

Solution. Let the length of the longer side of the garden be x m.

Then, breadth of the garden = $\frac{180}{x}$ m.

Total length of three sides of the garden = $x + \frac{180}{x} + \frac{180}{x}$

$\therefore \quad 39 = x + \frac{180}{x} + \frac{180}{x}$

$\Rightarrow \quad 39 = \frac{x^2 + 360}{x} \Rightarrow x^2 - 39x + 360 = 0$

$\Rightarrow \quad x^2 - 24x - 15x + 360 = 0$

$\Rightarrow \quad x(x - 24) - 15(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x - 15) = 0$

$\Rightarrow \quad x = 24$ or $x = 15$

If $x = 24$, breadth $\frac{180}{24} =$ m $= 7.5$ m

If $x = 15$, breadth $= \frac{180}{15}$ m $= 12$ m

Hence dimensions of the garden are 24 m by 7.5 m or 15 m by 12 m.

24. Reshma was asked to think of a natural number. She was asked to multiply its square by 5 then adds 8 and then subtract half the original number and write the final answer. She wrote 503. Find the number she thought of.

Solution. Let the required number be x.

Square of $x = x^2$

Half of the original number $= \frac{x}{2}$

$\therefore \quad 5x^2 + 8 - \frac{x}{2} = 503$

$\Rightarrow \quad \frac{10x^2 - x}{2} = 495 \Rightarrow 10x^2 - x - 990 = 0$

$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 + 4 \times 10 \times 990}}{2 \times 10} = \frac{1 \pm \sqrt{39601}}{20}$

$= \frac{1 \pm 199}{20}$

$\Rightarrow \quad x = \frac{1 + 199}{20}$ or $x = \frac{1 - 199}{20}$

$\Rightarrow \quad x = 10$ or $x = -9.9$

$\Rightarrow \quad x = 10$

[Rejecting $x = -9.9$, since x is a natural number]

Hence, the required number is 10